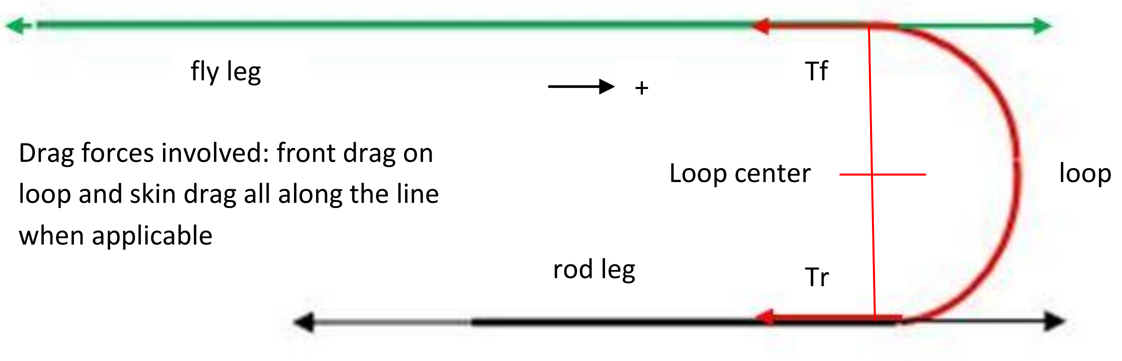
Equations for fly line motion and their use

(1D variable mass systems)

Here is a graphic summarizing the conditions of the exercise:



We divide the system into three subsystems (red, the loop; green, the fly leg and black the rod leg). We now apply the basic equation of momentum (p) variation in time for a variable mass system which is, for one single direction (originally this relationship applies to vectors):

dp/dt = external forces +u*dm/dt

dm is the mass entering or leaving the system, u is its corresponding (horizontal) speed.

For the loop, dp/dt = d/dt(loop mass*travel speed) which is (Vflyleg+Vrodleg)/2. External forces are tension at the ends of the loop and drag forces (D). We have to consider different tensions at loop ends (Tf and Tr) since nothing can tell us they are identical a priori. Tf is related to the fly leg, and Tr is related to the rod leg.

$$\frac{d\left(mloop * \frac{Vf+Vr}{2}\right)}{dt} = -(Tf+Tr) - D + \frac{dm}{dt} * \frac{Vf-Vr}{2} - \frac{dm}{dt} * \left(-\frac{Vf-Vr}{2}\right)$$

Since dm/dt = rholine*((Vf-Vr)/2)^2 we end up with:

$$\frac{d\left(mloop * \frac{Vf+Vr}{2}\right)}{dt} = 2 * \rho_{line} * ((Vf-Vr)/2)^2 - (Tf+Tr) - D$$

The contribution of each dm at ends are additive since the sign of both speed and dm change. Consequently we can see that tensions vary if the line accelerates or decelerates (d(Vf+Vr/2)/dt), if the line is tapered (d(mloop/dt), rholine), and if there is drag (D).

Incidentally, if we consider that the loop travels at constant speed, then d(Vf+Vr)/2/dt=0. If we consider there is no drag then D=0. And if we consider that tension T is the same at loop ends, we find the classic tension formula for a string or a string shooter:

$$0 = 2 * \rho line * V^2 - 2T$$

$$T = \rho line * V^2$$

Here V is the tangential / rotation speed of the loop ((Vf-Vr)/2). From this equation we can see that this formula represents an oversimplification of the situation studied here, which is already a simplification of reality. It is because such assumption was wrong that I restarted the modeling from scratch, since it was impossible to simulate reality (a nearly stall of the loop in case of pull back) at all with it.

You need to consider angular momentum (AM) if you want to identify tension at both ends of the loop. The basic AM equation is:

$$d/dt(J*\omega) = \text{external torques} + \omega\ dj/dt$$

J is the MOI of the loop, ω is its rotation speed around its center (see graphic at the beginning).

This time the contributions of both ends are opposite (same angular speed ω, opposite momentum of inertia change represented by dj), canceling each other and we end up with another equation which allows calculating Tf-Tr:

$$d/dt(mloop\ *\ R\hat{}2\ *\omega)\ =\ Torque\ from\ loop\ mass - Torque\ from\ drag - (Tf - Tr) * R$$

In the case of a level line (dm_{loop} =0) and a perfect half circle shape for the loop (no morphing which means dR = 0) we have:

$$mloop * R^2 * \frac{d\omega}{dt} = -Torque\ from\ drag + Torque\ from\ loop\ mass - (Tf - Tr) * R$$

Note that Rdω/dt = (Vf-Vr)/2

Assuming dR=0, this equation gives the difference in between tensions values and with the former equation involving their sum (horizontal momentum variation of the loop) we can calculate each of them, and then use the corresponding relationships to solve the equations for legs.

If m_{loop} varies (taper, then dm_{loop}/dt differs from 0) or if R varies (morphing), the situation offers greater complexity.

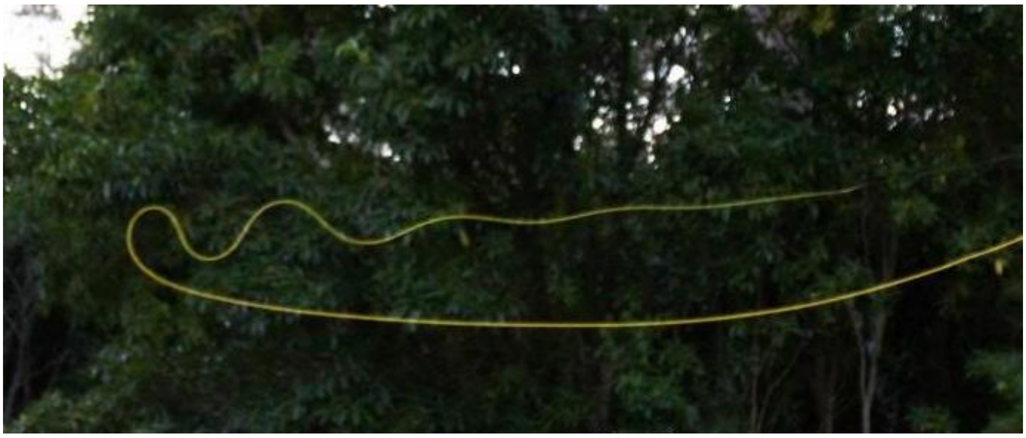
Without solving the general case (dR different from zero), we can see the trends with this expression:

$$2m_l\frac{dR}{dt} = -\frac{Dragtorque}{R} + \frac{loopmass\ torque}{R} - (Tf - Tr) - m_l\frac{d(Vf-Vr)/2}{dt} - (Vf - Vr)/2 * dm_l/\text{dt}$$

“Everything else being equal”, you can see that the term linked to the variation of the tangential/rotation speed of the loop has a negative sign so acceleration induced by a PB should correspond to a reduction trend of the loop size. It is more complex than that since there are interactions in between terms but you have a trend. There are other factors affecting loop size. For example, the vertical components of the front drag are squeezing the loop in the vertical direction, making the loop pointing. The skin drag forces mostly affect the upper part of the loop and that tends to make it asymmetrical, with potentially a larger length for the half bottom loop (the nose of the loop is somehow lifted).

One can also apply momentum variation equations in the vertical direction. In that case the AM equation which suggests a morphing of the loop (dR term) may be solved. You have to consider that the rod leg is inclined (sag, rod tip position) and that creates new parameters (angles between legs and loop) which transform an already delicate problem into a nightmare for Excel spreadsheets. Up until now, gravity influence is known: all will end on the ground and the analysis of rod leg sag can tell us what is happening. The sag is also sensitive to line acceleration and as soon as sag increases, it tells us that the loop decelerates and that tension is vanishing within the rod leg. Then it is up to the caster to correct this situation (e.g. pull back) if necessary.

There is an analogy with the string shooter which is interesting. There were a number of publications last year which confirm the importance of air drag and give information on the possibility of seeing a DN. The DN can occur for a string shooter if there is a discontinuity in tension somewhere around the loop. In that case waves coming from the fly leg side (e.g. fly shuttering) can be locked at loop level, exhibiting a stable pack of waves, like this one for example:



Let's see the equations for legs:

For the fly leg (mf is the mass of the fly leg and we skip the mass of the fly here, but it is included in my model, in fact you just add it to mf) we have the following relationship

$d/dt(mf*Vf) = Tf - \text{drag forces (fly, line skin drag)} + Vf\ dm/dt$

Be careful here since the "u" speed is Vf, and by developing we get:

$mf\ dVf/dt + Vf\ dmf/dt = Tf - \text{dragforces} + Vf\ dmf/dt$

dmf being negative for the fly leg (losing mass)

We can then simplify (Vf dm/dt is on both sides) and find the classic equation for the fly leg:

$mf\ dVf/dt = Tf - \text{drag forces}$

Ok then, Tf "pulls on the fly leg". We replace Tf by the value found with the methodology described above and we can get dVf/dt, which we can integrate numerically to get Vf.

For the rod leg (mr is the mass of the rod leg) we have:

$mr\ dVr/dt + Vr\ dmr/dt = Tr - F\ \text{(external force at tip)} - \text{drag (skin on the rod leg)} + Vr\ dmr/dt$

dmr being positive for the rod leg (getting mass)

Vr is the speed at which dmr enters the rod leg, and again we can simplify to end up with:

mr dVr/dt = Tr - F – line skin drag force (if the rod leg has some horizontal speed)

One has to define F for the purpose of the simulation. For example, if the cast is tethered then F = Tr; and if it is untethered then F =0. If there is a pullback you need to define a suitable F function. I usually use a sine wave one.

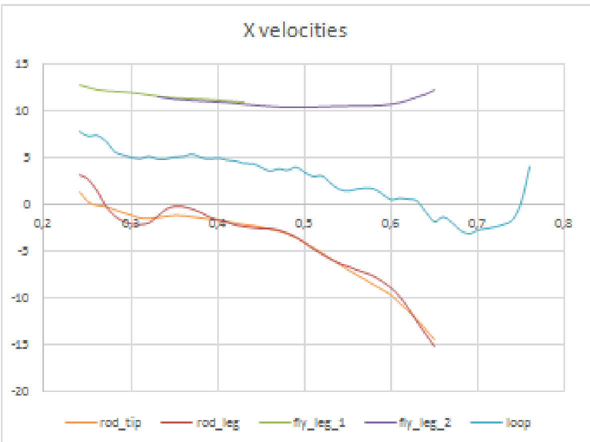
Further analysis

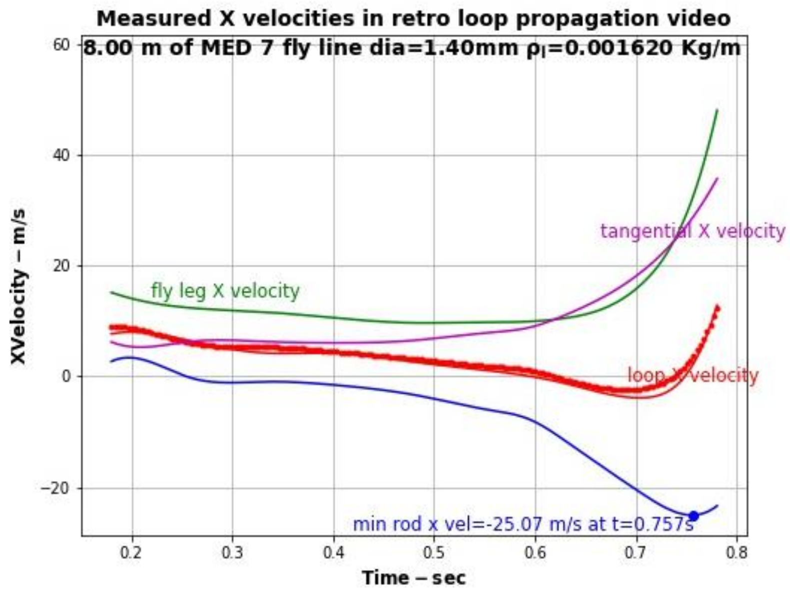
The use of AM equation allows evaluating how tension can vary along the loop; you have to calculate the variation of momentum for a defined tiny piece of line all along the loop. The key point to capture is that there is an effect of the acceleration of the loop rotation speed which cannot be ignored. In fact a PB or a check increases the angular momentum of the loop which affects the fly leg after a short while (a question of milliseconds). It was the subject of a hard discussion with Vince, but now that I did my way across tricky equations on my own (no one suggested a track for better understanding, James was mentioning AM but no more), I told Vince I was coming closer to his views. I do not despair to find a context where the tension at loop nose is very low (he thinks it may reach zero, just wait & see, I may find a compression one of these days). That was before realizing that tension discontinuity was possible.

Applications:

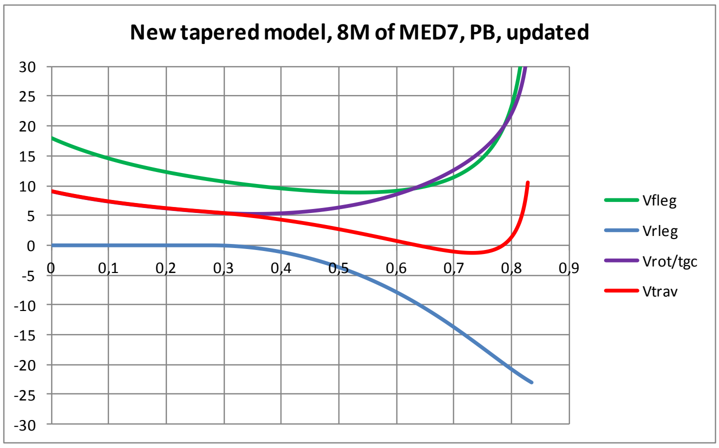
This has been an instructive exercise since it helped me identifying bugs in the model, and particularly a stupid introduction of weight instead of mass as a multiplier of the loop rotation / tangential acceleration.

Here is an illustration of a pullback simulating a cast recorded by Gordy with 8 meters of a MED 7, without leader. The analyses of the video with Tracker were performed respectively by Dirk and Gordy.

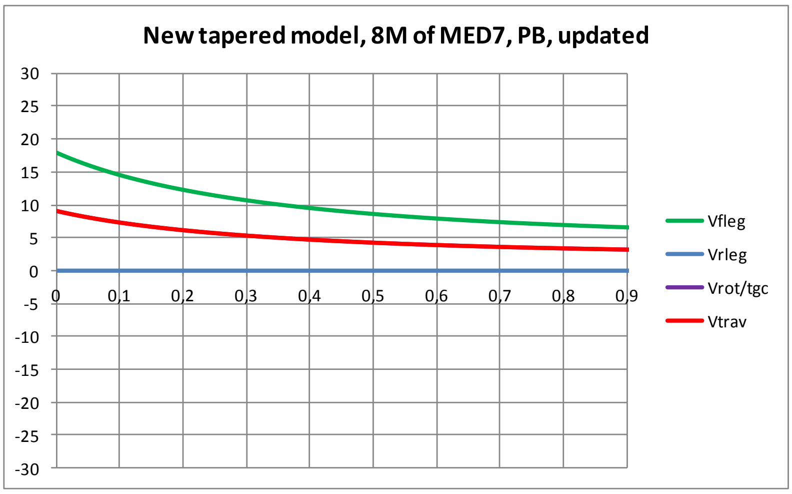




Now let's see the simulation. The aim is not to have a perfect fit in time and speeds but to capture the variations (I have used the same color code as Gordy):

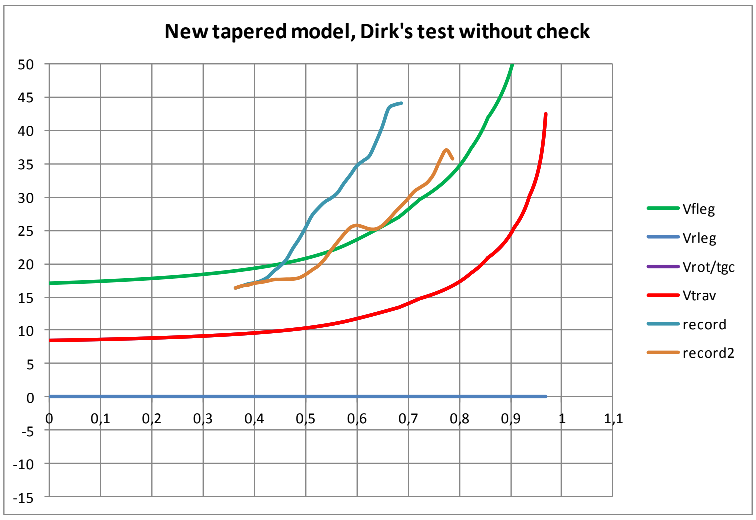


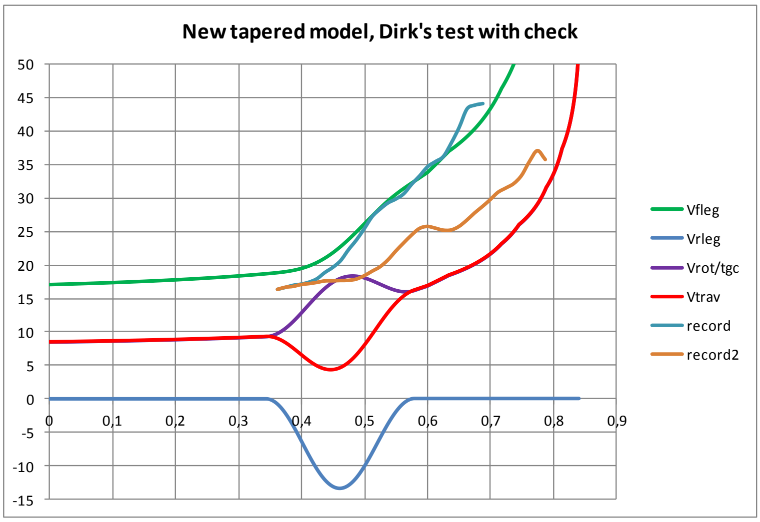
Sounds fine to me, you can see that the loop stalls and slightly go backwards (red curve goes below the zero level) before restarting forward and accelerate. Below is the same case without the PB action.



Note: there is no leader attached to the line.

Now here is another example with not so good match in between the simulation and the video record. In this case there is a leader attached to the line.





There are many unknowns for simulation, about a dozen. To try matching the video it was clear I had to change the skin drag coefficient for the leader. I really struggled with it since loop size was very important and finally I never succeeded in defining a set of suitable parameters. The cast occurred in specific atmospheric conditions (altitude, temperature, and wind) but for the time being I did not succeed in finding an improved compromise.

Not surprisingly, there are limitations for a simulation. Video analysis does not seem to be that easy by the way, so this type of matching exercise is pretty risky.

Nevertheless the basic equations in such simplified conditions are helpful. Loop morphing is taken out from the game but this is the price to pay for a simple and rather straightforward approach. Now there cannot be any miracle in the reality to model comparison.

Daniel, 1/11/2020