Measuring fly rod "Swingweight"

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1 Introduction

The term "swingweight" (or "swing weight") is well established in many sports which use a bat, club or racquet. In those sports, swingweight means the moment of inertia around a predetermined axis. We use the term swingweight to mean the moment of inertia of a single handed fly-rod around an axis at the butt of the rod. This standardizes usage and distinguishes swingweight from the moment of inertia around any other axis – particularly around the center of mass.

Moment of inertia (MOI) is resistance of an object to rotational acceleration. Greater MOI requires more torque to achieve the same rate of angular acceleration. So, the greater the MOI of a fly-rod the more force (torque) is needed to cast or manoeuvre that rod.

MOI is strongly dependent on mass distribution and distance from the axis of rotation. The dependence on distance from the axis is quadratic, so MOI increases with the square of distance to the axis of rotation. The following will generally be true:

- Long rods will have higher MOI than shorter rods of similar build and/or mass.
- Mass in the tip of the rod is much more important than the mass in the lower part of the rod (reel seat and grip).
- Rods with heavy blanks will have higher MOI than rods with light blanks.

This note is organized as follows. In Sec. 2 we present a method to calculate realistic swingweight values for multi-piece fly-rods. Sec. 3 gives a "cook-book recipe" for measuring and calculating fly-rod swingweight. Sec. 4 gives sample findings for 3 and 4 piece rods and we conclude in Sec. 5. For interested readers, we have included more detailed calculations in Appendix A.

2 Fly-rod MOI

In this section we present a method to estimate the moment of inertia of a multi piece single handed fly-rod. As noted above the MOI will be strongly dependant on the mass distribution in the rod, this information is not easily available. We therefore make two simple assumptions which allows us to estimate MOI based on simple non destructive measurements.

1. The mass density in each section of the rod decreases linearly with distance from the butt end of the section.

2. To account for the reel seat and grip we assume that the mass center of the butt section without reel seat and grip, is in the same position on the butt section as for the section above relative to the length of the sections[1].

Under this assumption we calculate the swing weight of the rod by measuring the length of the whole rod, the length of each section and distance from the butt end of each section to the mass center of the sections. (More detailed calculations and explanations are given in Appendix A.)

The moment of inertia I_{sec} for *each rod section* is then:

$$I_{\text{sec}} = m_{\text{sec}} l_{\text{sec}}^2 \left(\frac{x_{\text{cm}}}{l_{\text{sec}}} - \frac{1}{6} \right) - m_{\text{sec}} x_{\text{cm}}^2 + m_{\text{sec}} d_{\text{sec}}^2 \tag{1}$$

where m_{sec} is the mass of the section, l_{sec} is the length of the section, x_{cm} is the distance from the bottom (thick) end of the section to the balance point (mass center) and d_{sec} is the distance from the rod butt to the balance point for that section. When measuring or estimating d_{sec} make sure you take the overlap in the ferrules into account. We do this by assuming that the overlap is the same in all the ferrules, and equal to the length difference between the assembled rod and the sum of all the section lengths divided by the number of ferrules. If we now number the sections from butt to tip (used in subscripts bellow) we get the following for a 4 section rod for the distances d_{sec} in (1):

$$d_1 = \frac{x_{\text{cm2}}}{l_2} l_1 \tag{2}$$

$$d_2 = l_1 + x_{\text{cm2}} - \frac{1}{3}\Delta l \tag{3}$$

$$d_3 = l_1 + l_2 + x_{\text{cm3}} - \frac{2}{3}\Delta l \tag{4}$$

$$d_4 = l_1 + l_2 + l_3 + x_{\text{cm4}} - \Delta l \tag{5}$$

where Δl is the difference between the sum of all section lengths and the length of the assembled rod ($\Delta l = l_1 + l_2 + l_3 + l_4 - l_{\text{rod}}$). The rationale for (2) is given bellow.

Special care should be taken for the butt section of assembled rods since the reel seat adds a significant mass to the end of the section. This affects the mass distribution of the section, moving the mass center towards the butt, but is relatively unimportant for the overall moment of inertia for the rod. In order to get a better estimate for the butt section we suggest the following approach:

[1] For equal section lenght this is the same as assuming that the mass centers are at the same position.

1. Assume that the mass center of the blank is the same as for the section above relative to the length of the section. We call this x_{bcm} and it's generally given by:
$$x_{\mathrm{bcm}} = \frac{x_{\mathrm{cm2}}}{l_2} l_1. \tag{6}$$
2. Assume that the reel seat and grip can be represented by a thin uniform cylinder of length $l_{\mathrm{rg}} = 16$ cm. We also assume that that the mass density is uniform such that the mass center of the reel and grip is at $x_{\mathrm{rgcm}} = \frac{l_{\mathrm{rg}}}{2}$.
3. Estimate the mass of the seat and grip using this formula:
$$m_{\mathrm{rg}} = m_1 \frac{x_{\mathrm{cm1}} - x_{\mathrm{bcm}}}{x_{\mathrm{rgcm}} - x_{\mathrm{bcm}}} \tag{7}$$
4. Estimate the "effective mass" of the butt section blank m_{b1}:
$$m_{\mathrm{b1}} = m_1 - m_{\mathrm{rg}} \tag{8}$$
5. Estimate the moment of inertia for the seat and grip (I_{rg}):
$$I_{\mathrm{sg}} = \frac{1}{3} m_{\mathrm{rg}} l_{\mathrm{rg}}^2 \tag{9}$$
and the moment of inertia for the blank (I_{b1}) using m_{b} and x_{bcm} in (1) and add them together to get the "corrected" moment of inertia for the butt section ($I_1 = I_{\mathrm{rg}} + I_{\mathrm{b1}}$).

When you have calculated the moment of inertia for each section just sum them together to get the moment of inertia (swingweight) for the whole rod.

We recommend that the calculations are automated in a script, web form or a spreadsheet to make it less error-prone and ensure consistency between calculations.

3 Measure procedure

The measuring procedure is as follows: You need a tape measure, a good set of scales ($\frac{1}{10}$ g or better resolution), a hard edge on which to balance the rod and rod sections, and a multi-piece single-handed fly rod.

1. For each section, measure and write down the total section length, mass and distance from lower end of the section to the balance point (x_{cm}).
2. Do the same for the fully assembled rod.
3. Use the formula given in (1) to calculate the moment of inertia for each rod section with respect to an axis at the butt end of the reel seat. Pay attention to the fact that you should compensate for the overlap at the ferrules in the calculation (see example for a 4 pc. rod on page 2).

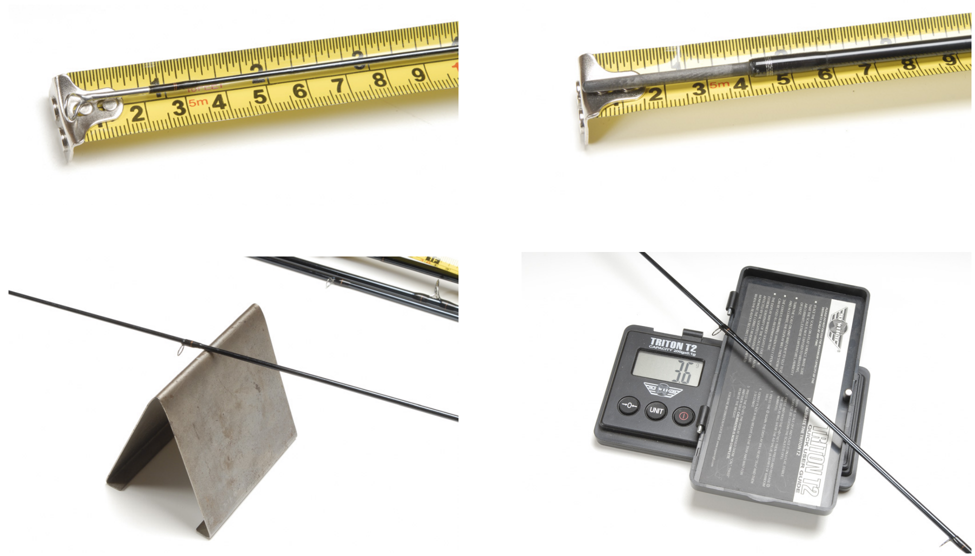

Figure 1: *Pictures illustrating parts of the measure procedure. How the tip is measured, how spigot ferrules are treated, how the balance point (mass center) of a section is located and weighting of a rod section.*

4. If you are measuring a finished rod (not a blank) use the procedure given at the end of the previous section (page 2) to compensate for the mass of the reel seat and grip and calculate the moment of inertia for the butt section of the rod.

5. Sum the moment of inertia for each section to get the moment of inertia (swingweight) for the whole rod.

Figure 1 show some of the elements in the measuring procedure, how tips and spigot ferrules are handled and how to find the balance point (mass center) of the rod sections.

4 Examples

The end result of the calculations can be understood in the following way. The swingweight is the resistance to angular acceleration when the rod is rotated around an axis at the butt of the rod. Think of the number as the resistance you would feel if you have the mass (indicated in your measured I_s) attached to the tip of an imaginary 1 m long massless stick. . .

Note that the result is quite sensitive to the accuracy of each measurement be as careful and exact as possible. If we assume that the uncertainty in the measured lengths are ~ 1 mm and the uncertainty in the measured masses are ~ 0.1 g we find that the uncertainty in the estimated swingweight I_s is in the order of ± 1 g m^2.

Dan Craft FT 905-4 Blank				
Sec. #	Mass (g)	Lenght (m)	Mass Center (m)	I_s (gm^2)
1	23.4	0.715	0.305	3.11
2	17.2	0.72	0.315	17.51
3	7.1	0.72	0.31	19.89
4	2.9	0.71	0.28	15.58
whole blank	50.6	2.74	0.83	56.1

Sage Z-axis 590-4				
Sec. #	Mass (g)	Lenght (m)	Mass Center (m)	I_s (gm^2)
1	68.9	0.72	0.16	3.53
2	15.9	0.72	0.31	15.80
3	8.4	0.72	0.32	23.40
4	3.7	0.72	0.33	20.36
whole rod	96.9	2.73	0.51	63.1

Orvis Helios 905-4 midflex				
Sec. #	Mass (g)	Lenght (m)	Mass Center (m)	I_s (gm^2)
1	45.7	0.73	0.19	3.16
2	13.6	0.72	0.30	13.71
3	7.9	0.72	0.33	22.91
4	4.3	0.72	0.32	24.03
whole rod	71.5	2.75	0.63	63.8

Table 1: *Example data for a fly-rod blank and two rods. Showing the weight, length and position of mass centers (measured from "the lower end" of the section) for each section (section 1 is the butt and section 4 is the tip), and the fully assembled blank/rod.*

In Table 1 and Table 2 measured data and calculated swingweights (I_s) for a random collection of rods are listed.

Swingweight is about the force needed to rotate a rod, the force need to achieve a required speed and then stop the rod. Swingweight may also be used as an index of precision or delicacy. Distance casting requires rapid acceleration it also requires a powerful lever/spring so swingweight should be considered along with stiffness (ERN).

5 Conclusion

We have developed a practical method with which to approximate the MOI of a fly rod. The measurements are relatively simple and sufficiently accurate, and the calculation can be automated.

Swingweight for single handed fly-rods, defined as the MOI for an axis at the rod butt, offers a representative and meaningfully method for comparing rods. Results can be expressed as a single figure which is representative of

Make	Model	# pcs.	Mass (g)	Length (m)	I_s (g m^2)
CTS	AF One 8' #3 (custom)	4	68.4	2.44	34.7
Echo2	596FW 9'6" #5	4	113.5	2.90	105.4
Echo2	510FW 10' #5	4	115.1	3.05	118.6
Greys	GRXi 9' #5/6	3	94.5	2.75	69.2
Guideline	LeCie 9' #5	4	95.5	2.752	66.8
Hardy	Angle TE 9' #5	4	94.0	2.75	71.1
Hardy	Swift 9' #5	3	86.7	2.745	65.0
Orvis	Helios Tipflex 9' #5	4	70.9	2.75	62.9
Orvis	Helios Tipflex 9' #8	4	87.9	2.75	81.6
Sage	SLT 590-4	4	91.2	2.73	65.5
Sage	SLT 7100-4	4	113.6	3.04	117.4
Sage	Z-Axis 590-4	4	96.7	2.73	63.1
Scott	S3 905-4	4	99.8	2.76	76.2
Targus	GB Pro. 908-4	4	105.6	2.76	80.0
Thomas & Thomas	PA 905-3	3	98.1	2.75	74.8
Winston	BIIx 905-4	4	84.9	2.747	64.2

Table 2: *Data and calculated "swingweights" I_s for a selected set of rods.*

caster experience, far more representative than simple rod mass.

Swingweight is the sum of section MOI. Graphed against length of section center of mass to butt, section MOI gives valuable insight into the effect of mass distribution and with the right type of data, into the effect of choices made when building a rod.

6 Acknowledgements

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A Explaining the model

In this section we will give some general comments on moment of inertia. And we will walk you through a more careful description of the model we use to calculate fly-rod swingweight.

The moment of inertia of a rod can generally be written as:

$$I = Aml^2 \tag{10}$$

Where A is a constant which depends on the mass distribution, m is the mass and l is the length of the rod. For a fly rod, the swingweight (which we have defined as: The moment of inertia with rotation around an axis perpendicular to the rod at the butt) is given by:

$$I_{\mathrm{s}} = \int_0^l \mu(x)x^2 dx \tag{11}$$

where l is the length of the fly rod, $\mu(x)$ is the linear mass density (mass per unit length) at distance x from the rod butt ($x = 0$ at rod butt).

Further if one knows the moment of inertia for an axis through the mass center (I_{cm}) one can calculate the moment of inertia around *any* axis parallel to the mass center axis through the *parallel axis theorem*:

$$I_{\mathrm{s}} = I_{\mathrm{cm}} + m\ x_{\mathrm{cm}}^2 \tag{12}$$

This can be used to calculate the moment of inertia around *any parallel axis* once we know the moment of inertia around *one axis* and the position of the *mass center (x_{cm})*. So if we know the moment of inertia around $x = 0$ (I_{s}) we can calculate the moment of inertia around the mass center (I_{cm})

$$I_{\mathrm{cm}} = I_{\mathrm{s}} - m\ x_{\mathrm{cm}}^2 \tag{13}$$

A.1 The model

In practice the problem is that we *don't know* the mass distribution ($\mu(x)$) along the rod and we don't know the moment of inertia around the center of mass. In order to get around this problems we introduce the assumption that the mass density of each rod section μ_{sec} is linear with the distance from the butt end of the section (thick end) x:

$$\mu_{\mathrm{sec}}(x) = a + bx \tag{14}$$

where a and b are constants which we can find by using:

$$m_{\mathrm{sec}} = \int_0^{l_{\mathrm{sec}}} \mu_{\mathrm{sec}}(x)dx \tag{15}$$

$$x_{\mathrm{cm}} = \frac{1}{m_{\mathrm{sec}}} \int_0^{l_{\mathrm{sec}}} \mu_{\mathrm{sec}}(x)x dx \tag{16}$$

where m_{sec} is the mass of the section, l_{sec} is the length of the section and x_{cm} is the position of the balance point (mass center) of the section with respect to the butt end. When the section mass m_{sec}, the length of the section l_{sec} and the position of it's center of mass x_{cm} are known we can show that:

$$a = \frac{m_{\text{sec}}}{l_{\text{sec}}}\left(1 - \frac{6}{l_{\text{sec}}}\left(x_{\text{cm}} - \frac{l}{2}\right)\right) \tag{17}$$

$$b = \frac{12 m_{\text{sec}}}{l_{\text{sec}}^3}\left(x_{\text{cm}} - \frac{l_{\text{sec}}}{2}\right) \tag{18}$$

which is valid for $x_{\text{cm}} \in [\frac{l_{\text{sec}}}{3}, \frac{2l_{\text{sec}}}{3}]$ (since we can't have negative mass densities).

From (14), (17) and (18) we can calculate the moment of inertia (I) for the section around an axis perpendicular to the blank at $x = 0$ and $x = x_{\text{cm}}$ (I_{cm})

$$\begin{aligned} I &= \int_0^l \mu_{\text{sec}}(x) x^2 dx \\ &= m_{\text{sec}} l^2 \left(\frac{x_{\text{cm}}}{l} - \frac{1}{6}\right) \end{aligned} \tag{19}$$

$$I_{\text{cm}} = m_{\text{sec}} l^2 \left(\frac{x_{\text{cm}}}{l} - \frac{1}{6}\right) - m_{\text{sec}} x_{\text{cm}}^2 \tag{20}$$

where the latter equation is given by the parallel axis theorem.

With this in mind we can now calculate the moment of inertia I_{sec} with respect to an axis of rotation at the butt of the rod for *each rod section* using (20) and (12):

$$I_{\text{sec}} = I_{\text{cm}} + m_{\text{sec}} d_{\text{sec}}^2 \tag{21}$$

$$= m_{\text{sec}} l_{\text{sec}}^2 \left(\frac{x_{\text{cm}}}{l_{\text{sec}}} - \frac{1}{6}\right) - m_{\text{sec}} x_{\text{cm}}^2 + m_{\text{sec}} d_{\text{sec}}^2 \tag{22}$$

where d_{sec} is the distance from the rod butt to the balance point for that section. When measuring or estimating d_{sec} make sure you take the overlap in the ferrules into account.

A.2 The butt section

The butt section of assembled rods need special attention. Since the reel seat and grip add and concentrate mass at the lower end of the section we can not use the mass center and linear mass density assumption directly. We compensate for this in the following manner:

We assume that the reel seat and grip can be represented by a 16 cm long ($l_{\text{rg}} = 16$ cm) thin uniform cylinder of mass m_{rg}. The mass center (x_{rgcm}) of this cylinder is then 8 cm from the butt ($x_{\text{rgcm}} = \frac{l_{\text{rg}}}{2}$). For a "normal" trout rod the this will be inside the reel seat close to the cork grip.

Assume that the mass center of the butt section blank (x_{bcm}) is in the same position as for the section above (relative to the section length).

$$x_{\text{bcm}} = \frac{x_{\text{cm2}}}{l_2} l_1 \tag{23}$$

Since the mass center position doesn't seem to vary that much from section to section this is the best guess we can make (see data for blank in Table 1).

Based on the two assumptions above we estimate the mass of the reel seat and grip m_{rg} and the blank m_{b1} by using the known (measured) mass center of the section (x_{cm1}):

$$m_1 x_{\text{cm1}} = m_{\text{rg}} x_{\text{rgcm}} + m_{\text{b1}} x_{\text{bcm}} \quad (24)$$

$$m_{\text{b1}} = m_1 - m_{\text{rg}} \quad (25)$$

$$m_{\text{rg}} = m_1 \frac{x_{\text{cm1}} - x_{\text{bcm}}}{x_{\text{rgcm}} - x_{\text{bcm}}} \quad (26)$$

The MOI for the reel seat and grip (I_{rg}) is then estimated by:

$$I_{\text{rg}} = \frac{m_{\text{rg}}}{l_{\text{rg}}} \int_0^{l_{\text{rg}}} l^2 dl \quad (27)$$

$$= \frac{1}{3} m_{\text{rg}} l_{\text{rg}}^2 \quad (28)$$

and the MOI for the blank is estimated using m_{b1} and x_{bcm} in Eq. (22). The total MOI for the butt section (I_1) is then the sum of the two contributions

$$I_1 = I_{\text{rg}} + I_{\text{b1}} \quad (29)$$

The total MOI for the rod is then the sum of the MOI for all the sections.