

Experimental investigations on the fly rod deflection

by construction assessor Tobias Hinzmann

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Preface

Physical investigations tend to be complicated, sophisticated and difficult to understand. It might also be a reason why physics often is not attributed to problems of the daily life when people discuss advantages or disadvantages of different tools or ideas. In fact our life is physics. Every object we use during the day, from the spoon in our cup of coffee to the subway that brings us to work is a physical object. The trajectories of sports equipment are especially promoted by physical investigations. From the bathing suite of a swimmer to the parachute of a sky diver, physics contributes to a far ranging optimization of such objects. However these studies are often performed in a laboratory that is only accessible for the scientist who conducts the optimization process. The open communication of the physical studies is often lacking – because it seems to be “too complicated”.

Myself and Tobias believe that this is not necessarily the case. It is moreover a question of communication to describe the physical properties of a system in an understandable way. The attribution of dynamic parameters to experimental data, an understandable description of each performed step, clear graphical representations of the trajectories and finally a mathematical “compromise” between a still valid estimation with mathematic expressions that are known from school instead of a complicated differential and integral calculus can do it.

With this work Tobias and I intend to convince the reader about the advantages of the flexible fly rod in comparison to an absolutely stiff fly rod by translating the dynamical properties of the fly rod into simplified mathematical expressions that are based on the mathematics as taught in the 9th or 10th class of school rather than higher mathematics on the university level. Each step is described semantically and the derivations are explained in a way that gives a real imagination of the dynamics of the flexible fly rod instead of presenting a pure final “value” after a long series of formulas which is hard to attribute to the physical object. In such way we believe that the study presented here might be more precious from a socio-scientific point of view than exact calculations based on theoretical physics that often cannot even be described by the theoretical physicist itself who often refers to the calculation process like a black box when asked what the result really means. This is a poor excuse for lacking phantasy how to correlate real social problems of the daily life to the power of exact physical treatment. We hope that the study shown here could be understood as a master example how to treat a socio-scientific problem by exact analysis to end up speculative discussions based on semantically controversies that lack any possibility to be proven. It turns out that there is no doubt that the transfer of effort spent on a flexible industrial fly rod into the acceleration of the tip and the fishing line must be much better than for an absolutely stiff fly rod.

I thank Tobias for the opportunity to contribute to a common problem of general interest. Our cooperation was a pleasure to me and so I didn't hesitate to translate his investigations into english.

Dr. Franz-Josef Schmitt

In the following I investigate physically/mathematically the influence of the fly rod deflection on the fly cast based on experimental data.

A) Background

The deflection of the fly rod is analyzed by a false fly casting sequence of me taken in summer 2012. It was registered by a commercial digital camera taking 30 frames per second. The single pictures of the sequence were analyzed with a computer program.

As fly rod the SAGE 586 RPL+ (2.65 m length) with suitable fishing line WF 5 F long belly was used.

For a better understanding of the influence of the deflection the properties of the used fly rod were compared with an absolutely stiff fly rod¹. The investigated rods therefore exhibit the following properties:

- 1.) The flexible fly rod has the stiffness of a fast action. The investigations are based on a fly rod of the type SAGE 586 RPL+ (fishing line class 5, ~2.65m length).
- 2.) The stiffness of the absolutely stiff fly rod is assumed to be infinite. (elastic modulus $\rightarrow \infty$), it shows no flexion at all during the fly cast.

The comparison of both rods is only possible if they are cast with the same angular speed (velocity of rotation). This assumption is used consequently for the following investigations.

For the sake of simplicity of the investigations some constraints are assumed for both types of rods:

- The rotational dynamics of the fly rod is analyzed. The parallel translation² is neglected due to it's small fraction of the overall dynamics.
- Both rods are cast with the same angular speed (rotational velocity).
- The rods do not carry mass.
- The deceleration and post-pulse oscillation of the rods is neglected.
- The initial and final positions of the rods are assumed to be at 40° and 140° relative to the horizontal line.
- A longer fishing line (about 22 meters including leader) is cast using a double haul.
- The tips of the rods accelerate the mass m of the elongated fishing lines.
- Both fly rods have the same length L .

The geometrical and dynamic numerical values are evaluated at three positions 40°, 90° and 140° and – if necessary – interpolated between these positions. The figures are used for visualization and are not fully true to scale. The whole investigation is presented aiming

¹ The indices „f“ and „s“ in the following denote: „f“ = „flexible fly rod“ and „s“ = „absolutely stiff fly rod“.

² The translational dynamics is separately analyzed at the end of this study.

to keep and present the physical relations as simple as possible³.

The results of the investigations are related to the analyzed fly cast sequence⁴. They are gathered by experiment and therefore carrying uncertainty. The uncertainty is discussed at the end of the study – currently it shall be stated that this fact does not question the general derivations of the study.

B) Geometrical investigations

In this section I investigate the geometry of the pathway described by the fly rod and the absolutely stiff rod during the fly cast.

B1) Fly rod

The following geometrical relations hold for the fly rod:

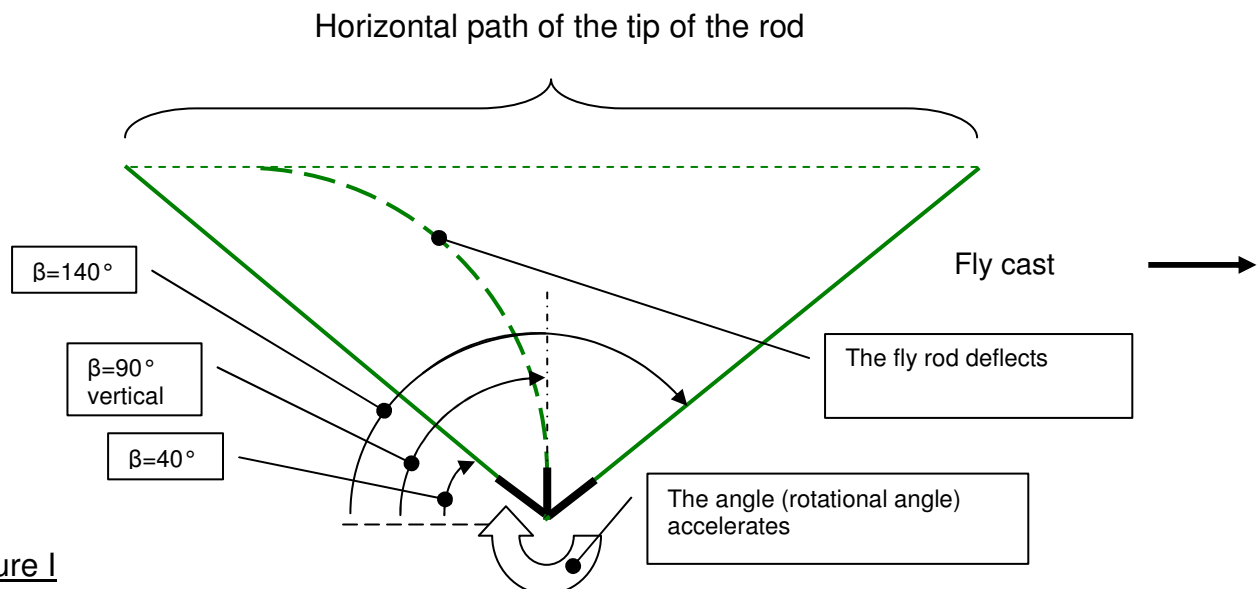


Figure I

The fly rod deflects. The experimental investigations show that the path of the tip of the rod is well described by a straight line (see Figure I).

1. Conclusion: The deflection of the fly rod leads to a dynamics of the rod tip moving along a straight line.

³ For example nor differential neither integral calculus is used. The values are calculated for the corresponding momentary positions of the fly rod and represent values that act in the fly rod at this time point exactly. Effects, that follow due to the dynamical interaction of the forces during the movement are included in the experimental data and therefore it is respected in the calculation that is based on the experiment. For that reason it can be assumed that this approach delivers realistic values.

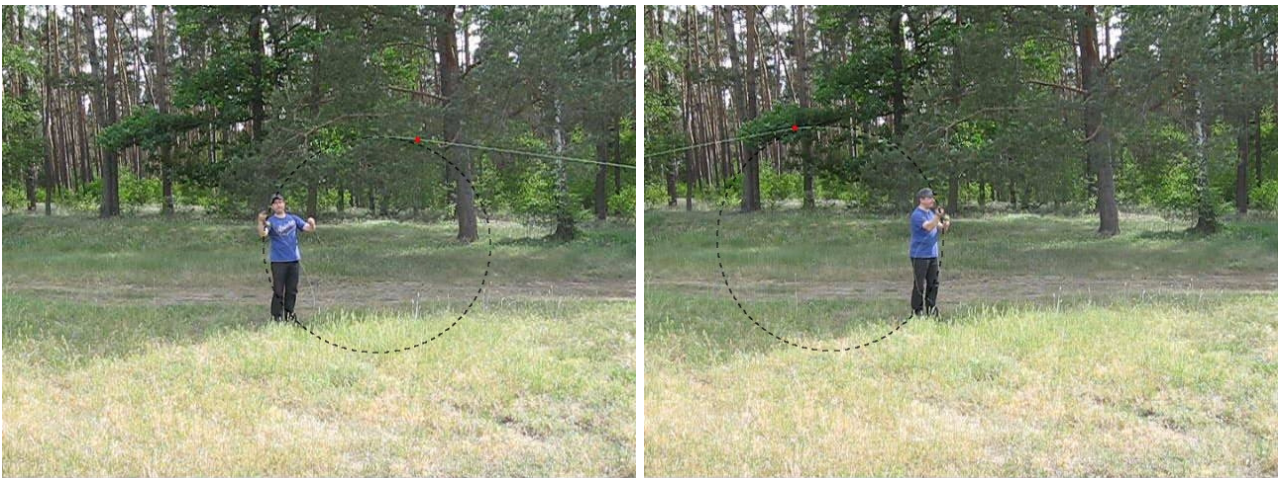
⁴ Single pictures of videos of other fly cast sequences show a nearly identical flection of the fly rod – although when casting the fly rod faster (SAGE TCR) or slower. The difference mainly results in a different final velocity of the rod tip.

B1.1) properties of the deflection of the fly rod

The position of the rod tip has to be determined for the following investigations. The accurate analysis of the deflection of the fly rod is necessary for that purpose. My fly cast sequence sheds light onto this question:

Deflection in the 90° position (vertical position)

To determine the geometry of the flection of the fly rod especially two pictures of my sequence are analyzed in more detail:



These two pictures of the forward and backward fly cast show that the deflection of the fly rod is well described by a segment of a circle. The pictures also show that the length of the fly rod is approximately a quarter of the circumference, when the grip of the rod is hold in the 90° position approximately. For the length of the fly rod and the circumference we get:

$$U = \pi * d = 2 * \pi * r; L \cong \frac{1}{4} * U$$

with U= circumference; d = diameter of the circle; r = radius of the circle; L = length of the fly rod

As the length L of the fly rod measures about one quarter of the circumfence U in the vertical position 90° of the grip the radius calculates to:

$$U = 4 * L = 2 * \pi * r \rightarrow r = \mathbf{0.64 * L}$$

The deflection shortens the projection of the fly rod onto the vertical 90° position by

$$L - r = L - 0,64 * L = \mathbf{0.36 * L}$$

Figure II shows the geometrical relations of the fly rod deflection.

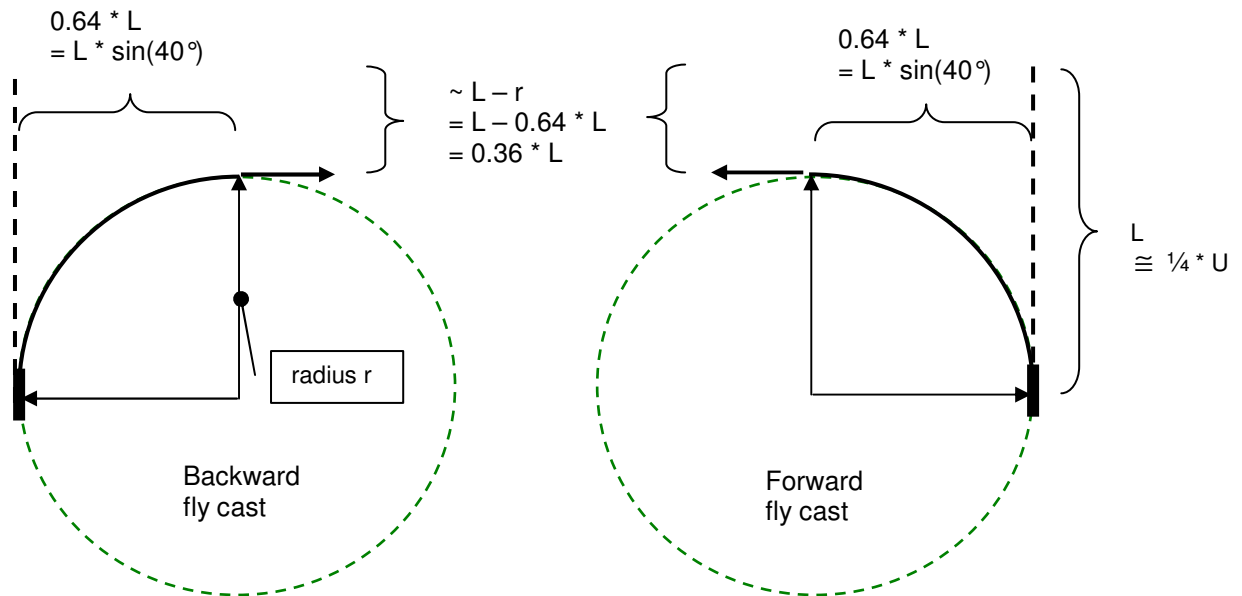


Figure II

2. Conclusion: The deflection of the fly rod leads to a shortened projection onto the vertical 90° position by about 1/3 (0.36*L).

Deflection in the 140° position (final position)

Again two single pictures of the fly cast sequence are analyzed to determine the geometry of the deflection of the fly rod.



The deflection of the fly rod in the 140° position is more complex than in the 90° position. The deflection approximates the form of the segment of an ellipse and cannot be described by a simple geometry. In the vertical 90° position the deflection was distributed equally over the whole length but in the 140° position the (upper) middle-section deflects much stronger than the other sections. It becomes visible that the tip of the rod

- continuously follows a straight path
 - Is localized approximately over the middle axis
- when the grip of the fly cast reached the final position (140° position)

B2) The absolutely stiff fly rod

The following geometrical relations result for the absolutely stiff fly rod:

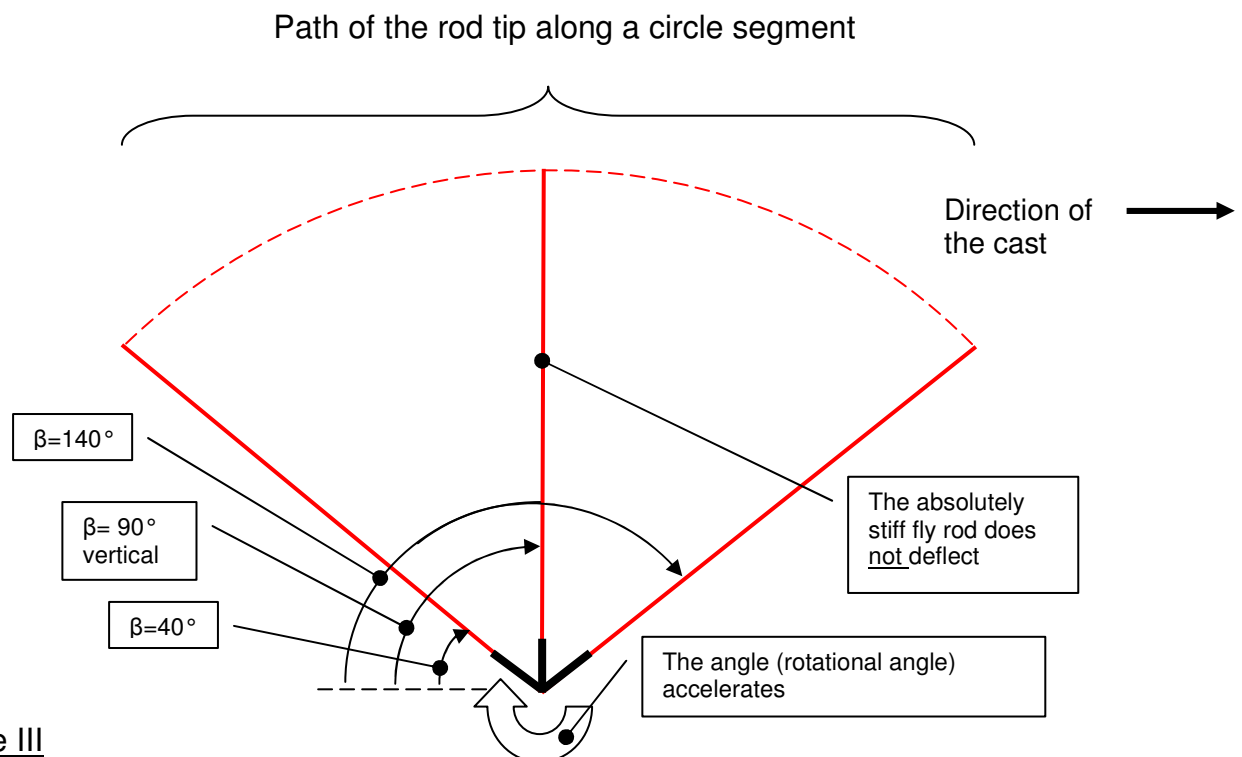


Figure III

The absolutely stiff fly rod does not deflect – even when force is applied. The path of the tip of the rod therefore describes the form of a convex circle segment (see Figure III)⁵.

B3) Path of the tip of the rod

The whole rotational angle α during the fly cast of both rods (flexible/ absolutely stiff) measures to:

$$\alpha = 140^\circ - 40^\circ = 100^\circ$$

The length of the path of the flexible fly rod's tip therefore calculates to

⁵ It is also possible to cast the absolutely stiff fishing rod during a small rotational angle in such way that the tip describes a straight line for a short distance. To gain comparable values it is assumed here that the tip of the absolutely stiff fly rod describes a real circle segment.

$$WRs(f) = 2 * L * \cos(40^\circ) = 2 * 0,77 * L = 1.54 * L$$

The path of the tip of the absolutely stiff fly rod describes a circle segment. It's circumference calculates according to the following formula:

$$WRs(s) = L * \pi * \frac{\alpha(deg)}{180^\circ} = L * \pi * 0.555 = 1.75 * L$$

Subsuming the previous investigations the following Figure compares the geometrical relations of the flexible and the absolutely stiff fly rod: (see Figure IV):

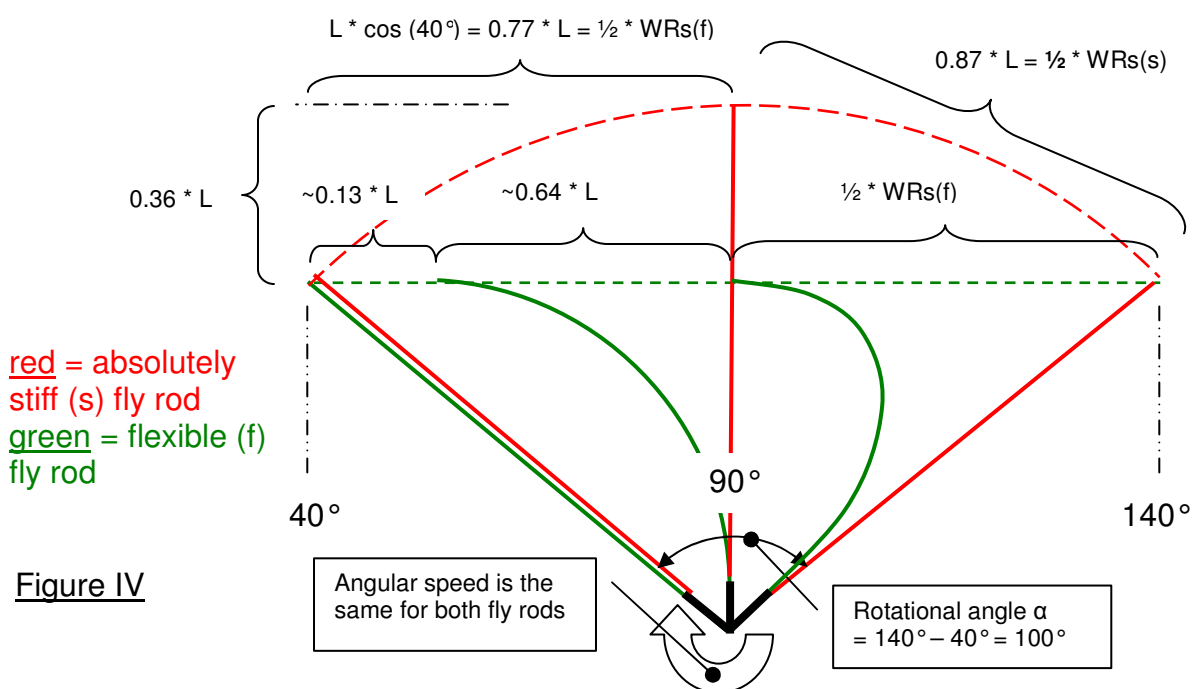


Figure IV

Position rotational angle / position of the rod's grip	40°	90° (vertical)	140° (end of rotation)	sum of the path length (without retraction / discharge)
Propagated path length of the tip of the flexible fly rod	0.0	$(0.13 - 0) * L = 0.13 * L$	$(0.77 - 0.13) * L = 0.64 * L$	$(0.13 + 0.64) * L = 0.77 * L = \frac{1}{2} WRs(f)$
Propagated path length of the tip of the absolutely stiff fishing rod	0.0	$(0.87 - 0) * L = 0.87 * L = \frac{1}{2} WRs(s)$	$(1.75 - 0.87) * L = 0.87 * L = \frac{1}{2} WRs(s)$	$(0.87 + 0.87) * L = 1.75 * L = WRs(s)$

B4) Impact directions

The rotational speed changing the rotational angle α , is accelerated in a similar way for both, the flexible and the absolutely stiff fly rod. But the introduced angular velocity is transferred differently to the tips.

B4.1) Direction of the velocity

Due to the deflection the tip of the flexible fly rod points in the same direction like the elongated fishing line during a long part of the path at the beginning of the fly cast. Therefore the fishing line is predominantly accelerated horizontally. In marked contrast the tip of the absolutely stiff fly rod does not point in the direction of the accelerated fishing line at any time because the tip accelerates the fishing line tangentially. (see Figure V).

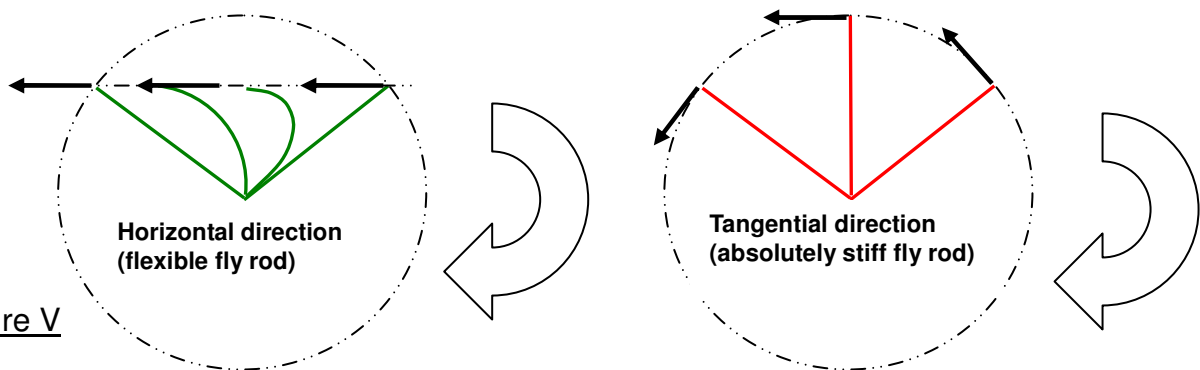


Figure V

3. Conclusion: *The tip of the flexible fly rod points into the direction of the elongated fishing line for a huge part of the path; it accelerates the fishing line in the horizontal direction. The tip of the absolutely stiff fly rod does not point into the direction of the elongated fishing line at any time; it accelerates the fishing line in the tangential direction along the segment of a circle.*

B4.2) Vectorial description of the impact directions

As the tips of both rods transfer the introduced rotational velocity differently to the fishing line, there follow different directions of the impact.

To be able to compare the flexible and the absolutely stiff fly rod, both rods do not only need to have the same rotational speed, but the tips have to cast the fishing lines into the same direction. Here the initially formulated constraint shall be concretized.

The fly rod shall be cast into horizontal direction so that the horizontal part of the velocity is in favor. The relations between the fractions and directions (vectors) of the velocity are shown in the following Figure VI:

Flexible fly rod (Final position)

Absolutely stiff fly rod (Final position)

As the tip of the fly rod moves along a straight line in the direction of the fishing line a decomposition is not necessary

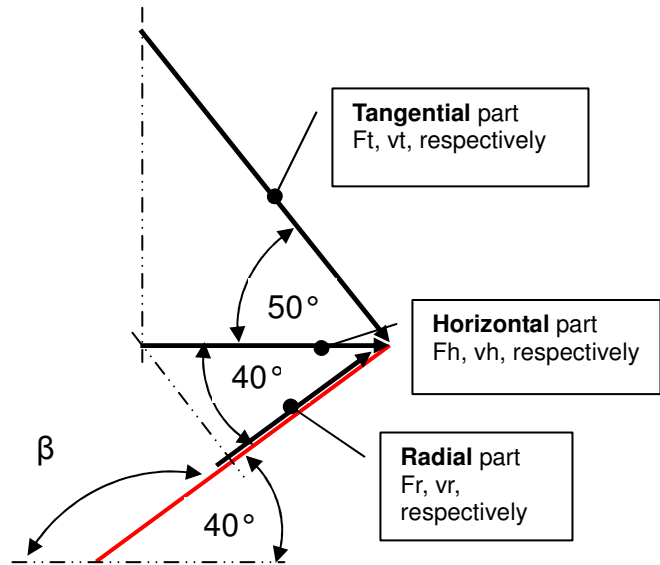
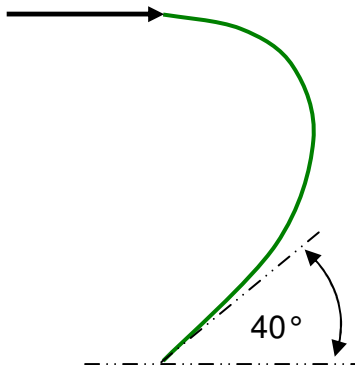


Figure VI

The relation between the tangential and the horizontal velocity of the absolutely stiff fly rod describes as follows:

$$v_h = \sin \beta * v_t$$

In the final position (140°- position) the horizontal fraction is:

$$v_h(140) = \cos(50) * v_t = \sin(140) * v_t$$

The previous vectorial relation shows, that the velocity components in both directions are correlated⁶. In the final position (140° position) the horizontal fraction calculates to

$$v_h(140) = \sin(140) * v_t = 0.643 * v_t$$

4. Conclusion: After a rotational angle α of 100° the horizontal part of the velocity of the tip reduces by a factor of 0.643 for the absolutely stiff fly rod in the final position. This factor reduces with rising rotational angle α and therefore the horizontal part of the velocity that can be reached with the absolutely stiff fly rod.

It is an advantage if the horizontal part of the velocity is as big as possible. This is only possible for the absolutely stiff fly rod if the rotational angle α is as small as possible.

⁶ And the horizontal part of the velocity can never become larger than the tangential part.

B4.3) Lever arm

If the rotational angle α changes, the tip of the fly rod moves the fishing line (mass) leading to a torque at the grip of the rod. This torque can be calculated with the formula

$$\text{torque} = \text{force} * \text{lever arm}$$

The length of the lever arm is approximately constant for the flexible fly rod during the whole path of the tip and corresponds to the radius of the circle segment $r = H_f = 0.64 * L$ (see section B1.1). For the absolutely stiff fly rod the length of the lever arm changes during the pathway of the tip and corresponds to the length of the rod L in the 90° position.

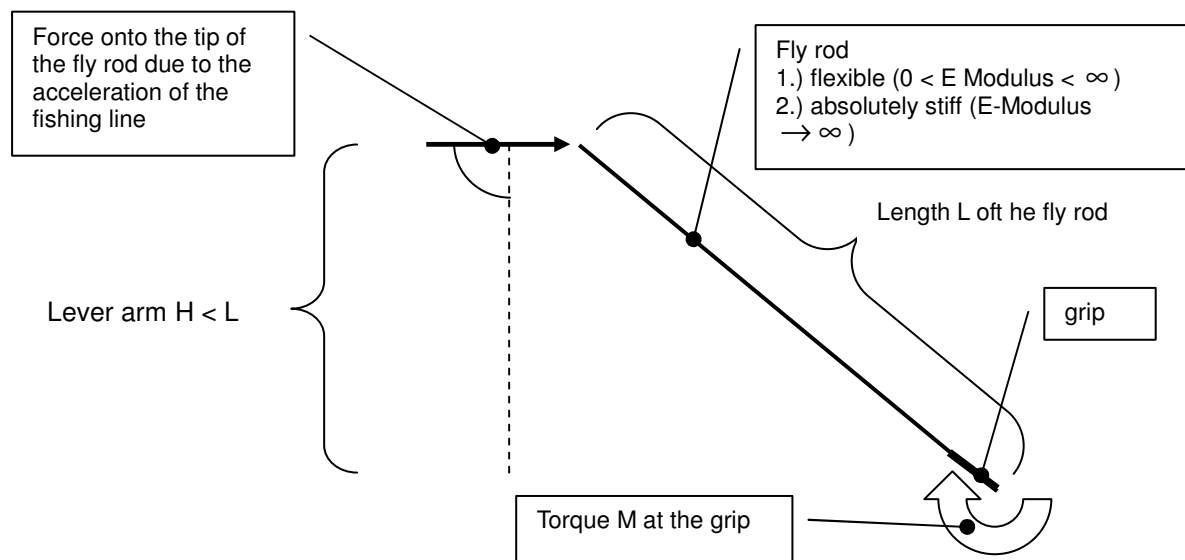


Figure VI

The torques are calculated later when the force that has to be spent by the fly caster is calculated.

C) Dynamical investigations

In this section I investigate the dynamics (time dependent relations) in more detail, describing the flexible and the absolutely stiff fly rod during the fly cast.

C1) Initiation of the stop, retraction/ discharge, respectively

In contrast to the absolutely stiff fly rod the deflection of the flexible fly rod additionally influences the process of the fly cast.

The single images of my fly cast sequence show the following typical signs for the initiation of a stop and the retraction/discharge, respectively:

- The linear part of the lower, strong section near the grip increases. The deflecting part of the fly rod decreases and moves into the direction of the tip of the fly rod.
- The hand at the grip of the rod has finished the rotation and just follows somehow into the direction of the fly cast

If the single pictures of my fly cast sequence are analyzed according to the aforementioned indications, then the retraction/discharge and the stop of the fly rod, respectively, initiate, when the tip has finished the first half of its path (see picture 3). The following five single pictures (one picture each 1/30 second) show the forward fly cast around the time point of the stop initiation:





picture 4



picture 5

The same can be assumed for my backwards fly cast, because the sequence of the fly cast and the properties of the deflection are virtually identically in both directions.

5. Conclusion: The stop, the retraction/discharge, respectively, of the flexible fly rod starts, when it's tip has completed about the first half (~50 %) of the path length (rotational path). The rod hand then has reached the final position at 140°.

The following Figure VII shows the relation between stop, the retraction/discharge, respectively, and the position of the tip of the rod.

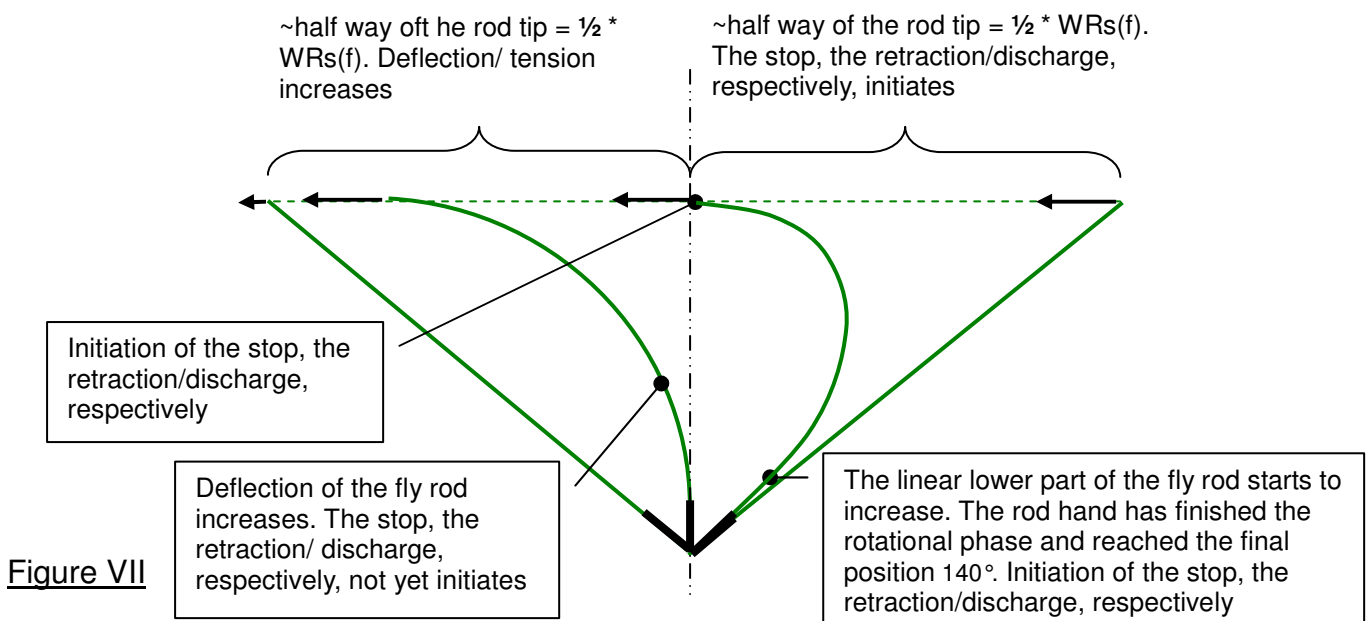


Figure VII

⁷ The fact alone, that the retraction/discharge takes half way of the rod tip shows, that it must not be neglected. Independent from the forthcoming conclusions it can be said at this point already that the fly caster has to control not only the increase of tension/deflection but also the release (retraction/discharge) during a long path length of the rod tip.

C2) Path-time-relations from the picture sequence of the fly cast

Now we can attribute single positions the rod tip reaches during its path to certain time points employing the geometrical relations found in section B) and the picture sequence. For that purpose the picture sequence of my forward fly cast is used in the following. The temporal interval measures 1/30 second. Due to the fact that the fly cast sequence and the deflection of the forward fly cast and the backward fly cast are virtually identical, the determined values also hold for the backward fly cast in a similar way.



pictures 1-3



pictures 4-6



pic 8:~vertical position 90°



pic 10:~initiation of retraction 140°



pictures 11-13

In the last picture, 13, the fly cast sequence is terminated already.

This can be seen from the following indications:

- In the previous picture 12 the fly rod is already nearly discharged, only the most upper part of the tip is slightly deflected
- The path of the fly rod shows downwards
- The fly rod has just passed the resting position
- The fishing line starts to exhibit a noose

Therefore the last picture is neglected for the determination of the duration of the fly cast⁸.

The times calculated from the picture sequence are shown in the following table:

Position rotational angle / position fly cast grip	90°	140°	Retraction/ discharge/stop until resting position
Number of pictures until the position of the rotational angle / rod grip is reached	8	10	~12
Overall time (duration)	8 / 30 = 0.2666s	10 / 30 = 0.3333s	12 / 30 = 0.40s
Differences	40° to 90°	90° to 140°	from beginning until the end of retraction/discharge
Time interval Δt (at 30 pictures per second) between the positions	$\Delta t = (8-0)/30 = 0.2666s$	$\Delta t = (10-8)/30 = 0.0666s$	$\Delta t = (12-10)/30 = 0.0666s$

6. conclusion: The tips of both rods take different time periods, to reach the final horizontal velocity. At the same rotational speed the fly cast of the flexible fly rod takes about 0.0666 seconds longer (the time period of the retraction/discharge) than the fly cast of the absolutely stiff fly rod.

C3) Determination of velocities from the picture sequence

During the single pictures of the fly cast sequence the tip and the elongated fishing line move with a velocity

⁸ If the last, 13th picture would also be respected, temporal values would be gathered that do not represent the reality. For example the velocity would decelerate during the retraction/discharge. Then the noose would already form during the retraction/discharge, which is not the case.

$$v = \frac{\Delta path}{\Delta time} = \frac{\Delta w}{\Delta t}$$

The velocities calculated in such way correspond to the velocities the rod tip reaches between the positions 40° to 90° as well as 90° to 140°. My forward fly cast was exemplarily chosen for the calculation, my backwards fly cast would deliver a comparable picture sequence.

C3.1) Velocities of the tip of the flexible fly rod

In the following table the velocities of the tip of the flexible fly rod are calculated:

Position of the rotational angle/ position of the grip	90°	140°	Retraction/discharge/ stop until resting position
Covered distance of the rod tip (flexible)	0.13 * L	0.77 * L	1.54 * L
Differences	40° to 90°	90° to 140°	from beginning until the end of retraction/discharge
Velocity vf of the rod tip $\Delta w/\Delta t$	$vf = (0.13-0.00)L / 0.2666s = 0.487L/s$	$vf = (0.77-0.13)L / 0,0666s = 9.61L/s$	$vf = (1.54-0.77)L / 0.0666s = 11.56L/s$

C3.2) Velocities of the tip of the absolutely stiff fly rod

For the same picture sequence the velocities of the rod tip of the absolutely stiff fly rod are calculated in the following table⁹. The horizontal part vs,h follows according to section B4.2)

Position of the rotational angle/ position of the grip	90°	140°	Retraction/discharge/ stop until resting position
Covered distance of the rod tip (absolutely stiff)	0.87 * L	1.75 * L	not the case
Differences	40° to 90°	90° to 140°	From beginning until the end of retraction/discharge
<u>Tangential</u> velocities vs,t of the rod tip	$vs,t = (0.87-0,00)L / 0.2666s = 3.26L/s$	$vs,t = (1.75-0.87)L / 0.0666s=13.063L/s$	not the case
<u>Horizontal</u> velocities vs,h of the rod tip	$vs,h=vs,t*\sin(90^\circ) = 3.26L/s * 1.00 = 3.26L/s$	$vs,h=vs,t*\sin(140^\circ) = 13.063L/s * 0.643 = 8.40L/s$	not the case

⁹ By using the same picture sequence it is guaranteed that both rods are analyzed with the same rotational speed.

The following Figure VIII shows the graph of the velocities of the tips of both rods¹⁰.

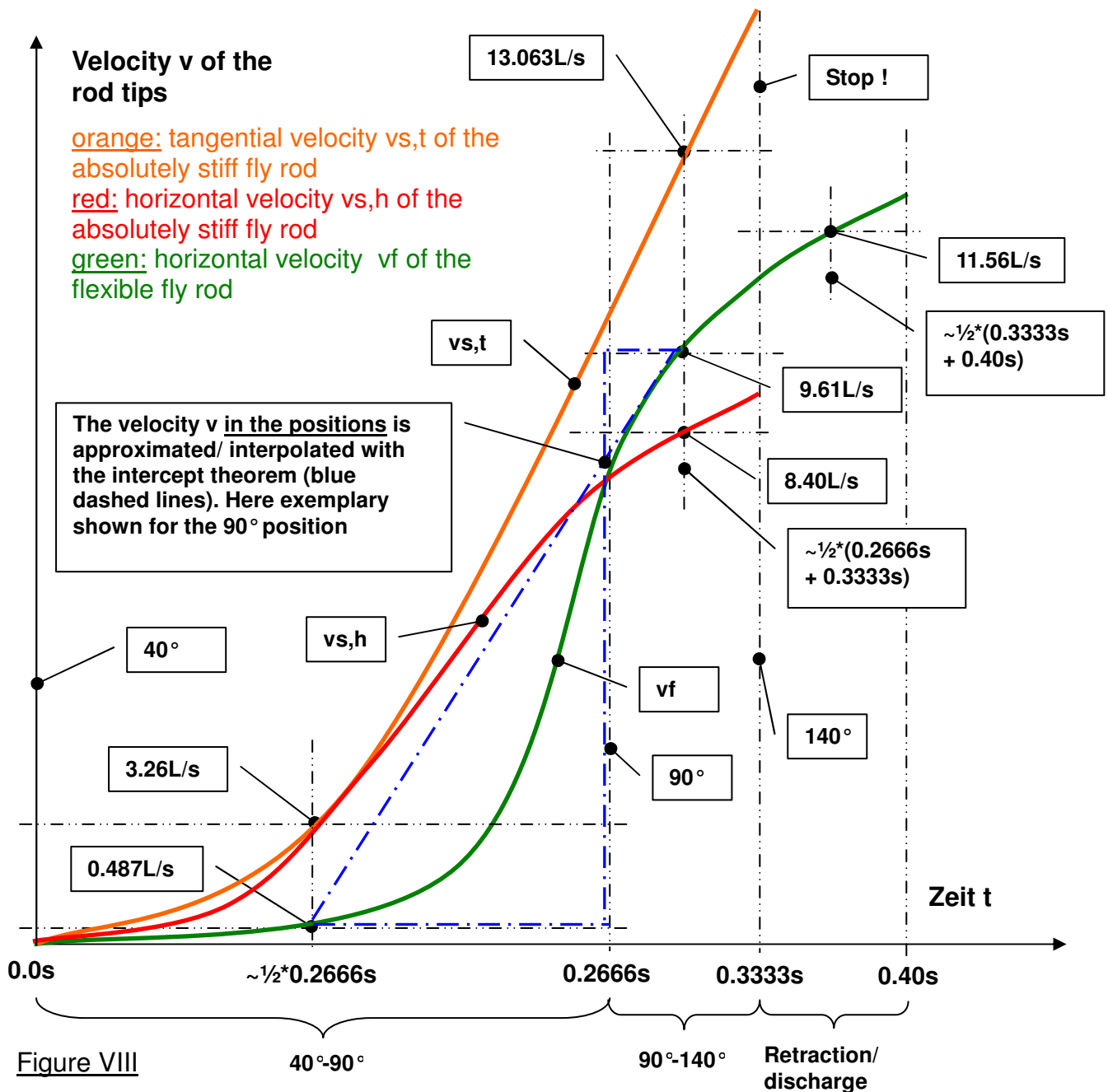


Figure VIII

¹⁰ The average velocities between the positions 40° to 90° and 90° to 140° are attributed to the middle time points for the subsequent investigations, that means: $v(40^\circ-90^\circ) \approx \frac{1}{2} * (0.2666s+0.00s)$, $v(90^\circ-140^\circ) \approx \frac{1}{2} * (0.2666s+0.3333s)$ und $v(\text{deflection}) \approx \frac{1}{2} * (0.3333s+0.40s)$. The determined time course of the rotational speed of my casting sequence deviates mainly initially from the linear correlation. Therefore the temporal assignment is not exact. The previously mentioned assignment is chosen for the sake of simplicity, because a more exact determination would be complex and not in proportion to the expected deviation, which is assumed to be neglect able for the undergone investigations – also because it annihilates most probable when comparing both fly rods.

The velocities direct in the positions 90°, 140° and at the end of the retraction / discharge are well approximated by the intercept theorem¹¹ in the following.

For the tip of the flexible fly rod the horizontal velocity v_f calculates to:

$$\frac{0.5 * ((0.2666 - 0) + (0.3333 - 0.2666))}{(9.61 - 0.487)} = \frac{0.5 * 0.2666}{v_f(90^\circ) - 0.487}$$

$$\rightarrow v_f(90^\circ) = \frac{0.5 * 0.2666 * 9.123}{0.1666} + 0.487 = 7.30 + 0.487 = \mathbf{7.787 \text{ L/s}}$$

$$\frac{0.5 * ((0.3333 - 0.2666) + (0.40 - 0.3333))}{(11.56 - 9.61)} = \frac{0.5 * (0.3333 - 0.2666)}{v_f(140^\circ) - 9.61}$$

$$\rightarrow v_f(140^\circ) = \frac{0.5 * 0.0666 * 1.95}{0.0666} + 9.61 = 0.975 + 9.61 = \mathbf{10.585 \text{ L/s}}$$

$$\frac{0.5 * (0.40 - 0.3333)}{(11.56 - 10.585)} = \frac{0.5 * (0.40 - 0.3333)}{v_f(\text{end}) - 11.56}$$

$$\rightarrow v_f(\text{end}) = \frac{0.5 * 0.0666 * 0.975}{0.0333} + 11.56 = 0.975 + 11.56 = \mathbf{12.535 \text{ L/s}}$$

For the tip of the absolutely stiff fly rod the horizontal velocity $v_{s,h}$ calculates to:

$$\frac{0.5 * ((0.2666 - 0) + (0.3333 - 0.2666))}{(8.40 - 3.26)} = \frac{0.5 * 0.2666}{v_{s,h}(90^\circ) - 3.26}$$

$$\rightarrow v_{s,h}(90^\circ) = \frac{0.5 * 0.2666 * 5.14}{0.1666} + 3.26 = 4.113 + 3.26 = \mathbf{7.373 \text{ L/s}}$$

$$\frac{0.5 * ((0.3333 - 0.2666) + (0.40 - 0.3333))}{(8.40 - 7.373)} = \frac{0.5 * (0.3333 - 0.2666)}{v_{s,h}(140^\circ) - 8.40}$$

$$\rightarrow v_{s,h}(140^\circ) = \frac{0.5 * 0.0666 * 1.027}{0.0333} + 8.40 = 1.027 + 8.40 = \mathbf{9.427 \text{ L/s}} = v_{s,h}(\text{end})$$

The comparison of the velocities $v(\text{end})$ and the initial velocities $v(40^\circ-90^\circ)$ of the tips of

¹¹ Intercept theorem: http://en.wikipedia.org/wiki/Intercept_theorem. For the example 90° the equations according to the intercept theorem denote to:

$(\frac{1}{2} t(40^\circ \text{ to } 90^\circ) + \frac{1}{2} t(90^\circ \text{ to } 140^\circ)) : (v(90^\circ \text{ to } 140^\circ) - v(40^\circ \text{ to } 90^\circ)) = \frac{1}{2} t(40^\circ \text{ to } 90^\circ) : (v(90^\circ) - v(40^\circ \text{ to } 90^\circ))$

the rods results in:

$$vf(\text{end}) / vs,h(\text{end}) = 12.53 / 9.43 = \mathbf{1.33}$$

$$vf(40^\circ-90^\circ) / vs,h(40^\circ-90^\circ) \approx 0.487 / 3.26 = \mathbf{0.15}$$

7. Conclusion: The tip of the fly rod has a final horizontal velocity which is about 33 % higher in comparison to the absolutely stiff fly rod. In marked contrast the tip of the flexible fly rod has only 15 % of the initial velocity in comparison to the absolutely stiff fly rod at the beginning of the fly cast (initial velocity)¹².

The comparison of the velocities of the tip of the flexible fly rod at the beginning and at the end of the stop, the retraction/discharge, respectively delivers:

$$vf(140) / vf(\text{end}) = 10.585 / 12.535 = \mathbf{0.84}$$

8. Conclusion: The tip of the flexible fly rod has more than 80 % of it's final velocity at the beginning of the stop/ retraction/ discharge¹³.

C4) Calculation of the accelerations

The acceleration is defined as the change of the velocity in time¹⁴. Now the acceleration can be calculated for each section from the previously calculated velocities:

$$a = \frac{\Delta \text{velocity}}{\Delta \text{time}} = \frac{\Delta v}{\Delta t}$$

For the tip of the flexible fly rod the acceleration a_f calculates to:

$$af1 = a(40^\circ-90^\circ) = \frac{0.487 - 0}{0.5 * (0.2666 - 0)} \frac{L}{s^2} = \frac{0.487}{0.1333} = \mathbf{3.65} \frac{L}{s^2}$$

¹² The introduced rotational speed is transferred into the velocity of the tip with retardation. The rotational speed that is spent at the beginning of the fly cast nearly completely leads to deflection of the fly rod (which causes a remarkable increase of the potential tension force) and not to increasing velocity of the tip. When the tip reaches approximately the vertical 90° position, the tip of the flexible fly rod has reached the horizontal velocity of the tip of the absolutely stiff fly rod and continues to accelerate. Finally the tip of the flexible fly rod exhibits a much larger horizontal velocity than the tip of the absolutely stiff fly rod at the end of the fly cast.

¹³ The fact that the tip of the flexible fly rod nearly reaches it's final velocity at the initiation of the stop/ the retraction/ discharge is often used as an argument, to underestimate the value of it's deflection. This view neglects that exactly due to the deflection the tip of the flexible fly rod reaches a remarkable higher final horizontal speed than the tip of the absolutely stiff fly rod and that the flexible fly rod distributes the velocity of the tip differently along it's path (see 7th conclusion). The different distribution of the velocity of the rod tip will be analyzed in more detail when the efficiency is considered.

¹⁴ The acceleration is the derivative of the velocity: $a = v'$.

$$af2 = a(90^\circ) = \frac{9.61 - 0.487}{0.5 * ((0.2666 + 0.3333) - 0.2666)} \frac{L}{s^2} = \frac{9.123}{0.1666} = \mathbf{54.76} \frac{L}{s^2}$$

$$af3 = a(90^\circ - 140^\circ) = \frac{10.585 - 7.787}{0.3333 - 0.2666} \frac{L}{s^2} = \frac{2.796}{0.0666} = \mathbf{41.98} \frac{L}{s^2}$$

$$af4 = a(140^\circ) = \frac{11.56 - 9.61}{0.5 * ((0.3333 + 0.40) - (0.2666 + 0.3333))} \frac{L}{s^2} = \frac{1.95}{0.0666} = \mathbf{29.23} \frac{L}{s^2}$$

$$af5 = a(\text{discharge}) = \frac{12.535 - 10.585}{0.40 - 0.3333} \frac{L}{s^2} = \frac{1.95}{0.0666} = \mathbf{29.23} \frac{L}{s^2}$$

For the tip of the absolutely stiff fly rod the acceleration¹⁵ as calculates to:

$$as1 = a(40^\circ - 90^\circ) = \frac{3.26 - 0}{0.5 * (0.2666 - 0)} \frac{L}{s^2} = \frac{3.26}{0.1333} = \mathbf{24.45} \frac{L}{s^2}$$

$$as2 = a(90^\circ) = \frac{8.40 - 3.26}{0.5 * ((0.2666 + 0.3333) - 0.2666)} \frac{L}{s^2} = \frac{5.14}{0.1666} = \mathbf{30.85} \frac{L}{s^2}$$

$$as3 = a(90^\circ - 140^\circ) = \frac{9.427 - 7.373}{0.3333 - 0.2666} \frac{L}{s^2} = \frac{2.054}{0.0666} = \mathbf{30.79} \frac{L}{s^2}$$

The following Figure IX shows the curves of the accelerations of the tips of both rods:

¹⁵ In the following the index „h“ for „horizontal“ is omitted. All accelerations refer to the horizontal direction.

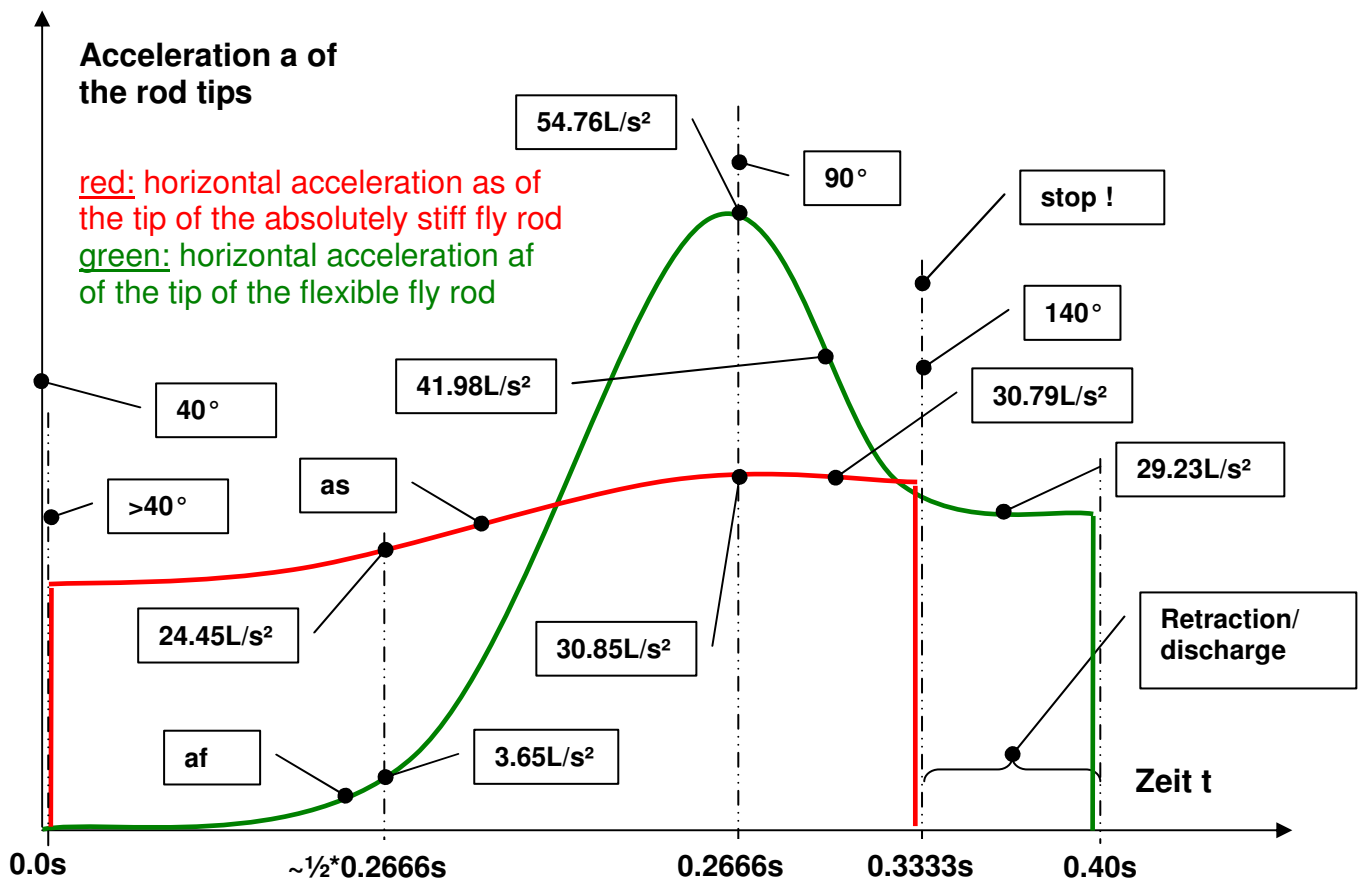


Figure IX

D) Calculation of the forces

In this section I investigate the forces in more detail, which are present during the fly cast of the flexible and the absolutely stiff fly rod: For the calculation of the forces Newton's second law is used:

$$\text{Force} = \text{mass} * \text{acceleration}; F = m * a$$

The mass m is represented by the weight of the elongated fishing line which pulls at the tip of the fly rod.

D1) The potential force of tension that acts on the tip

The tension on the fishing line (mass m) and the rotation of the own mass of the fly rod leads to its deflection. For the further investigations we assume:

- At the averaged time point between the 40° and the 90° position (~65°) the fly rod tip has not moved at all or covered a neglectable distance
- The retraction /discharge occurs from half the way of the fly rod tip

Both assumptions follow from the picture sequence (section C2). The assumptions lead to the following displacements of the tip of the flexible fly rod (see table):

Position Rotationswinkel / Stellung Rutengriff	40° to 90° ~1/2 of time	90°	140°	~1/2 time retraction/ discharge, stop, respectively	~ full time retraction/ discharge, stop, respectively
Covered distance of the rod tip (WRs) <u>with</u> deflection	~0.0L	0.13L	0.77L	~(0.77+0.5*0.77)*L = 1.155L	1.54 L
Covered distance of the rod tip along the axis of the rod (<u>without</u> deflection)	~0.5*0.77L = 0.385L	0.77 L	1.54 L	1.54 L	1.54 L
difference = displacement x = deflection	0.385 L	0.64 L	0.77 L	0.385 L	0.0 L
Difference of the displacement Δx = change of the deflection	(0.385-0)*L = <u>0.385 L</u>	(0.64- 0.385)*L = <u>0.255 L</u>	(0.77- 0.64)*L = <u>0.13 L</u>	(0.385-0.77)*L = <u>-0.385 L</u>	(0-0.385)*L = <u>-0.385 L</u>

9. Conclusion: The displacement measures the deflection of the fly rod. The displacement increases for more than until the 90° position is reached. The whole displacement introduced into the rotation (change of rotational angle) increases the deflection of the fly rod.

The whole potential energy that is stored in the fly rod is transferred to kinetic energy via the retraction/discharge due to the deflection¹⁶, because the fly caster does not continue the rotation and therefore does not infer more energy or work, respectively. As the velocities are known at the beginning and the end of the retraction/discharge, it is possible to calculate the potential energy.

$$\begin{aligned}
 E_{pot,(discharge)} &= \frac{1}{2} * m * v_{f(end)}^2 - \frac{1}{2} * m * v_{f(140^\circ)}^2 \\
 &= \frac{1}{2} * m * \left(12.535^2 \frac{L^2}{s^2} - 10.585^2 \frac{L^2}{s^2} \right) = \mathbf{22.54} \frac{m * L^2}{s^2}
 \end{aligned}$$

To get the potential tension force, the energy must be divided through the translated distance:

$$F_{pot,5} = F_{(discharge)} = E_{pot,(discharge)} / \text{Distance} = 22.54 / 0.77L$$

¹⁶ It is correct, to look at the energy $E = \frac{1}{2} m v^2$ to treat the problem according to the energy conservation law. The energy, the fly rod releases during retraction/discharge must have been spent by the fly caster before.

$$= \mathbf{29.27} \frac{m * L}{s^2} = F_{\text{pot,all}}$$

This tension force which acts during the retraction / discharge must have been spent by the fly caster. It can be estimated and approximately distributed by the known displacements x and Δx , respectively

Until the initiation of the retraction/ discharge, it follows:

$$F_{\text{pot},1} = 0.385L/0.77L * 29.27 = \mathbf{14.63} \frac{m * L}{s^2} = F_{\text{pot}}(40^\circ-90^\circ)$$

$$\Delta F_{\text{pot},1} = 0.385L/0.77L * 29.27 = \mathbf{14.63} \frac{m * L}{s^2}$$

$$F_{\text{pot},2} = 0.64L/0.77L * 29.27 = \mathbf{24.33} \frac{m * L}{s^2} = F_{\text{pot}}(90^\circ)$$

$$\Delta F_{\text{pot},2} = 0.255L/0.77L * 29.27 = \mathbf{9.69} \frac{m * L}{s^2}$$

$$F_{\text{pot},3} = 0.77L/0.77L * 29.27 = \mathbf{29.27} \frac{m * L}{s^2} = F_{\text{pot}}(90^\circ-140^\circ)$$

$$\Delta F_{\text{pot},3} = 0.13L/0.77L * 29.27 = \mathbf{4.94} \frac{m * L}{s^2}$$

$$F_{\text{pot},4} = 0.77L/0.77L * 29.27 = \mathbf{29.27} \frac{m * L}{s^2} = F_{\text{pot}}(140^\circ)$$

$$\Delta F_{\text{pot},4} \approx 0.00L/0.77L * 29.27 \approx \mathbf{0.00} \frac{m * L}{s^2}$$

With the initiation of the retraction / discharge it follows¹⁷:

$$F_{\text{pot}}(\text{discharge,initial}) = 0.77L/-0.77L * 29.27 = \mathbf{-29.27} \frac{m * L}{s^2}$$

$$\Delta F_{\text{pot}}(\text{discharge,initial}) \approx 0.00L/-0.77L * 29.27 = \mathbf{0.00} \frac{m * L}{s^2} \approx \Delta F_{\text{pot}}(140^\circ)$$

$$F_{\text{pot}}(\text{discharge,mid}) = 0.385L/-0.77L * 29.27 = \mathbf{-14.63} \frac{m * L}{s^2}$$

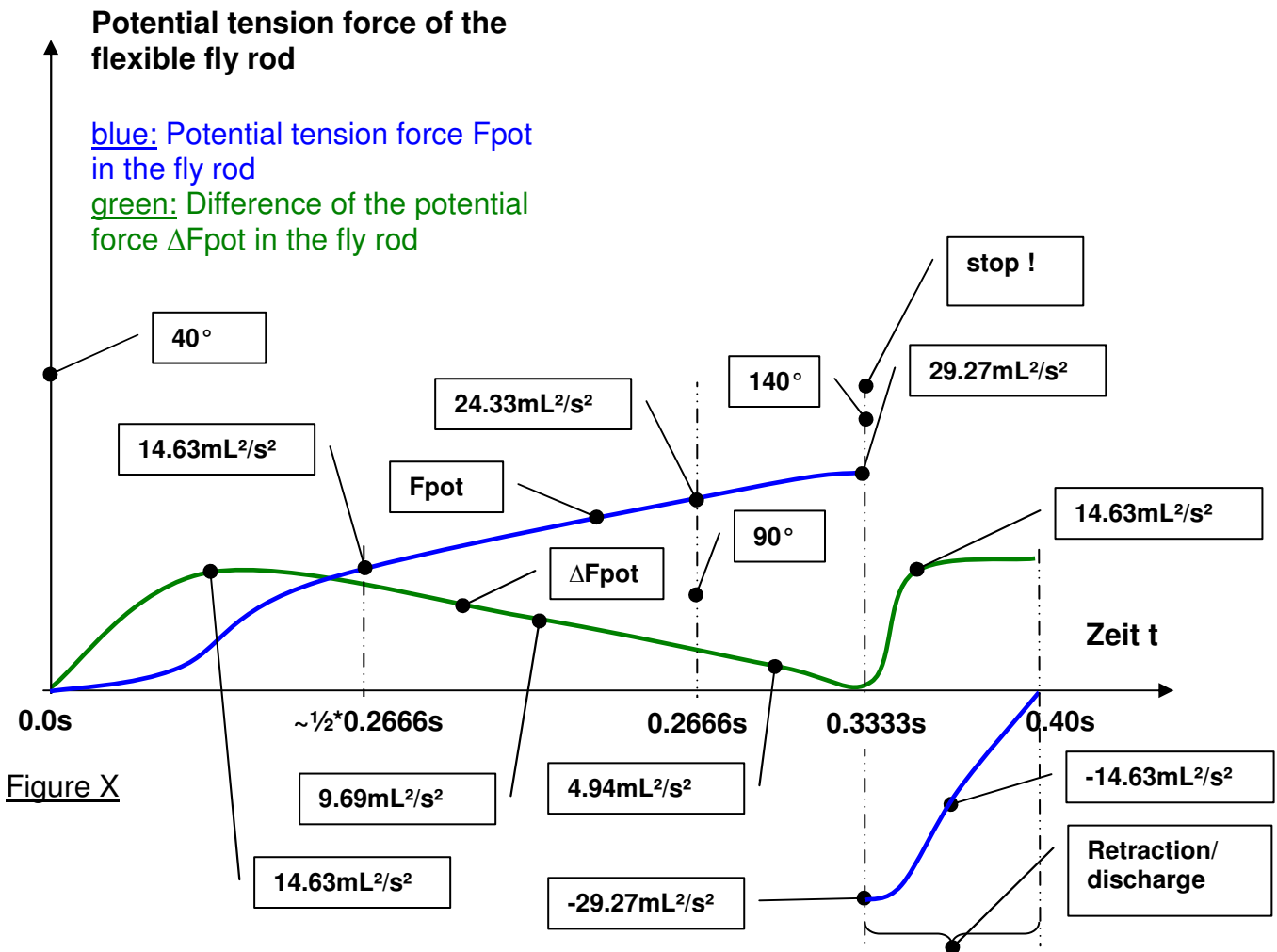
$$\Delta F_{\text{pot}}(\text{discharge,mid}) = -0.385L/-0.77L * 29.27 = \mathbf{14.63} \frac{m * L}{s^2}$$

$$F_{\text{pot}}(\text{discharge,end}) = 0.00L/-0.77L * 29.27 = \mathbf{0.00} \frac{m * L}{s^2}$$

$$\Delta F_{\text{pot, end}} = -0.385L/-0.77L * 29.27 = \mathbf{14.63} \frac{m * L}{s^2} = \Delta F(\text{discharge,end})$$

¹⁷ The direction of the retraction / discharge leads to a negative sign. Like for the elastic spring the path length which leads to the deflection (charging) and retraction (discharging) must have different signs.

The following Figure X shows the curve of the potential tension force of the flexible fly rod¹⁸.



The calculated course of the potential tension force shows a continuous change (the current results clearly point it out including the picture sequence in section C2). The continuous change of the potential tension force leads to an optimized transfer of velocity onto the fishing line. If the fly caster neglects this fact (and settles pressure points - „handshock“), the tip of the fly rod becomes uneasily and introduces waves as well as tailing loops into the fishing line. This leads to a reasonable loss of the transferred energy over the path length of the tension of the fishing line. The continuous change of the tension force during the duration of the fly cast helps to it's controlled formation and depletion

¹⁸ According to the initially stated constraints it is assumed that both fly rods do not carry mass. The fact that the deflection of the fly rod increases in spite of decreasing acceleration has to be attributed to the real existing mass of the rod. The approximately stated assignment of the potential tension force as well as the drawn conclusions are still valid, because the own mass leads to increasing forces onto the rod tip and therefore also the acceleration and the spent force equally increase. The experimental data uniquely show that the deflection of the fly rod increases until the stop, the initiation of the retraction/discharge, respectively ! For the calculation of the potential tension forces and the efficiency this effect can be taken into account in future work.

guiding the rod tip to run smoothly¹⁹.

D2) Forces acting on the tips of both rods

For the flexible fly rod also kinetic forces (acceleration forces) act next to the potential tension force (force of position). For the absolutely stiff fly rod only the kinetic force acts, because the rod does not deflect and therefore potential tension force cannot be stored.

The fact that the absolutely stiff fly rod cannot take up potential tension force leads to a different distribution of the forces in comparison to the flexible fly rod.

10. Conclusion: The initial rotation leads to increasing deflection for the flexible fly rod causing an increase of the potential tension force without an acceleration of the mass of the fishing line. For the absolutely stiff fly rod each initial movement immediately leads to an acceleration of the fishing line and the immediate need to spend force for it.

For the flexible fly rod it follows:

$$F = m * a + \Delta F_{pot}^{20}$$

$$F_{f0} \approx 0 = F(>40^\circ) \text{ (See 10. Conclusion)}$$

$$F_{f1} = m * 3.653 \frac{L}{s^2} + 14.63 \frac{m * L}{s^2} = \mathbf{18.28} \frac{m * L}{s^2} = F_f(40^\circ-90^\circ)$$

$$F_{f2} = m * 54.76 \frac{L}{s^2} + 9.69 \frac{m * L}{s^2} = \mathbf{64.45} \frac{m * L}{s^2} = F_f(90^\circ)$$

$$F_{f3} = m * 41.98 \frac{L}{s^2} + 4.94 \frac{m * L}{s^2} = \mathbf{46.92} \frac{m * L}{s^2} = F_f(90^\circ-140^\circ)$$

$$F_{f4} = m * 29.27 \frac{L}{s^2} + 0.0 \frac{m * L}{s^2} = \mathbf{29.27} \frac{m * L}{s^2} = F_f(140^\circ)$$

¹⁹ For a comparison of the controlled generation and release of potential tension forces the wings of an arch can be considered, because the potential tension forces should develop similarly in that case (There are differences of course, e.g. the fact that the fly rod releases the potential tension force starting from the initiation of the stop – the archer might retard this; another fact might be that in contrast to the wings of an arch the tip of the fly rod has already a relative speed with the initiation of the retardation/discharge). The archer continuously rises the tension of the arch wings aiming to let them accelerate the arrow with highest precision – also the fly caster should rise the tension of the fly rod as continuously as possible, aiming to let the tip of the fly rod accelerate the fishing line precisely. The arrow shall not oscillate during its flight to avoid the loss of energy – also waves in the fishing line should be avoided. The fly caster can and should not be investigated equally with an archer. But there are parallel effects regarding the impact of the potential tension force and their controlled generation and release, which can show the influence of the tension better than other comparisons. This comparison has made the impact of the potential tension forces more understandable for Dr. Schmitt (see acknowledgments) as he had personally stated.

²⁰ The fly caster has to spend the change of the potential tension force ΔF_{pot} . Only the change of a state needs force that performs work. For the sake of simplicity $\Delta F_{pot,1}$ is attributed to the position between 40° and 90° and $\Delta F_{pot,2}$ the position 90° . This assignment is more reliable because by this fact higher forces are assigned to the flexible fly rod in comparison to the alternative assignment by the average.

$$Ff5 = \mathbf{14.63} \frac{m * L}{s^2} = Ff(\text{discharge, end})$$

For the absolutely stiff fly rod it follows:

$$F = m * a; \Delta F_{\text{pot}} = 0;$$

$$Fs0 \approx \mathbf{24.45} \frac{m * L}{s^2} = Fs(>40^\circ) \text{ (see 10. conclusion)}$$

$$Fs1 = \mathbf{24.45} \frac{m * L}{s^2} = Fs(40^\circ - 90^\circ)$$

$$Fs2 = \mathbf{30.85} \frac{m * L}{s^2} = Fs(90^\circ)$$

$$Fs3 = \mathbf{30.79} \frac{m * L}{s^2} = Fs(90^\circ - 140^\circ);$$

D3) Forces (torques) at the grip of both fly rods

During the rotation of the fly rod a torque M acts on the grip, which has to be spent by the fly caster. The torque calculates as product of force and length of the lever.

The forces F have been calculated in section D2) and the lever in section B4). According to this the length of the lever arm of the flexible fly rod is virtually constant along the whole path of the rod tip. For the absolutely stiff fly rod the length of the lever arm in the 40°, 90° and 140° position is known. On the half way between these positions the length of the lever is approximately the algebraic average²¹.

For the flexible fly rod it follows:

$$Mf0(>40^\circ) \approx 0$$

$$Mf1(40^\circ - 90^\circ) = 18.28 * 0.64 L = \mathbf{11.70} \frac{m * L^2}{s^2}$$

$$Mf2(90^\circ) = 64.45 * 0.64 L = \mathbf{41.25} \frac{m * L^2}{s^2}$$

$$Mf3(90^\circ - 140^\circ) = 46.92 * 0.64 L = \mathbf{30.02} \frac{m * L^2}{s^2}$$

$$Mf4(140^\circ) = 29.27 * 0.64 L = \mathbf{18.73} \frac{m * L^2}{s^2}$$

²¹ Taking a shortened projection of the lever arm into account for the absolutely stiff fly rod leads to a calculation of the "lower limit" of the effort ("best case") because in the following also the diminished horizontal part only of the force is respected (see section B4). This treatment promotes the effort of the absolutely stiff fishing rod which would be higher in reality regarding the investigated rotational movement. This conservative treatment also pays attention to the fact that the tip of the absolutely stiff fly rod can be conducted along a straight line at least for a short distance (small rotational angle)

$$Mf5 = 14.64 * 0.64 L = \mathbf{9.36} \frac{m * L^2}{s^2} = M(\text{discharge, end})$$

For the absolutely stiff fly rod it follows:

$$Ms0(>40^\circ) \approx 24.45 * 0.64 L = \mathbf{15.65} \frac{m * L^2}{s^2}$$

$$Ms1(40^\circ-90^\circ) = 24.45 * \frac{1}{2} (0.64 + 1.0) L = \mathbf{20.05} \frac{m * L^2}{s^2}$$

$$Ms2(90^\circ) = 30.85 * 1.0 L = \mathbf{30.85} \frac{m * L^2}{s^2}$$

$$Ms3(90^\circ-140^\circ) = 30.79 * \frac{1}{2} (1.0 + 0.64) L = \mathbf{25.25} \frac{m * L^2}{s^2}$$

$$Ms4(140^\circ) \approx 30.79 * 0.64 L = \mathbf{19.70} \frac{m * L^2}{s^2}$$

11. Conclusion: The torque which has to be spent by the fly caster rises with the square of the length L of the fly rod²².

²² I sold my first fly rod measuring 10 feet (3.0 meter) with fishing line class 8 after short usage. It was so stiff that my wrist could not transfer the rotational movement without reasonable effort of force and started to hurt after short time in spite of the fact that this fly rod was "only" 10 % longer than the usual 9 feet (2.70 m). The fact that the force (torque) rises with the square of the length of the fly rod explains the reasonable increase of the additional force effort, which is significantly larger than the 10 % length increase of the fly rod. Since then I chose my few fly rods measuring 10 feet, accurately and paid attention to a „fast deep action“. During this action the fly rod deflects comparatively more in the lower segment. Therefore the lever arm shortens on the one hand and the deflection and the path of the retraction / discharge increases. This reduced the effort of force and such a fly rod finally still carries enough potential of retraction, to throw the fishing line far. Also this example shows that the deflection of the fly rod plays an important role for the effort of force !

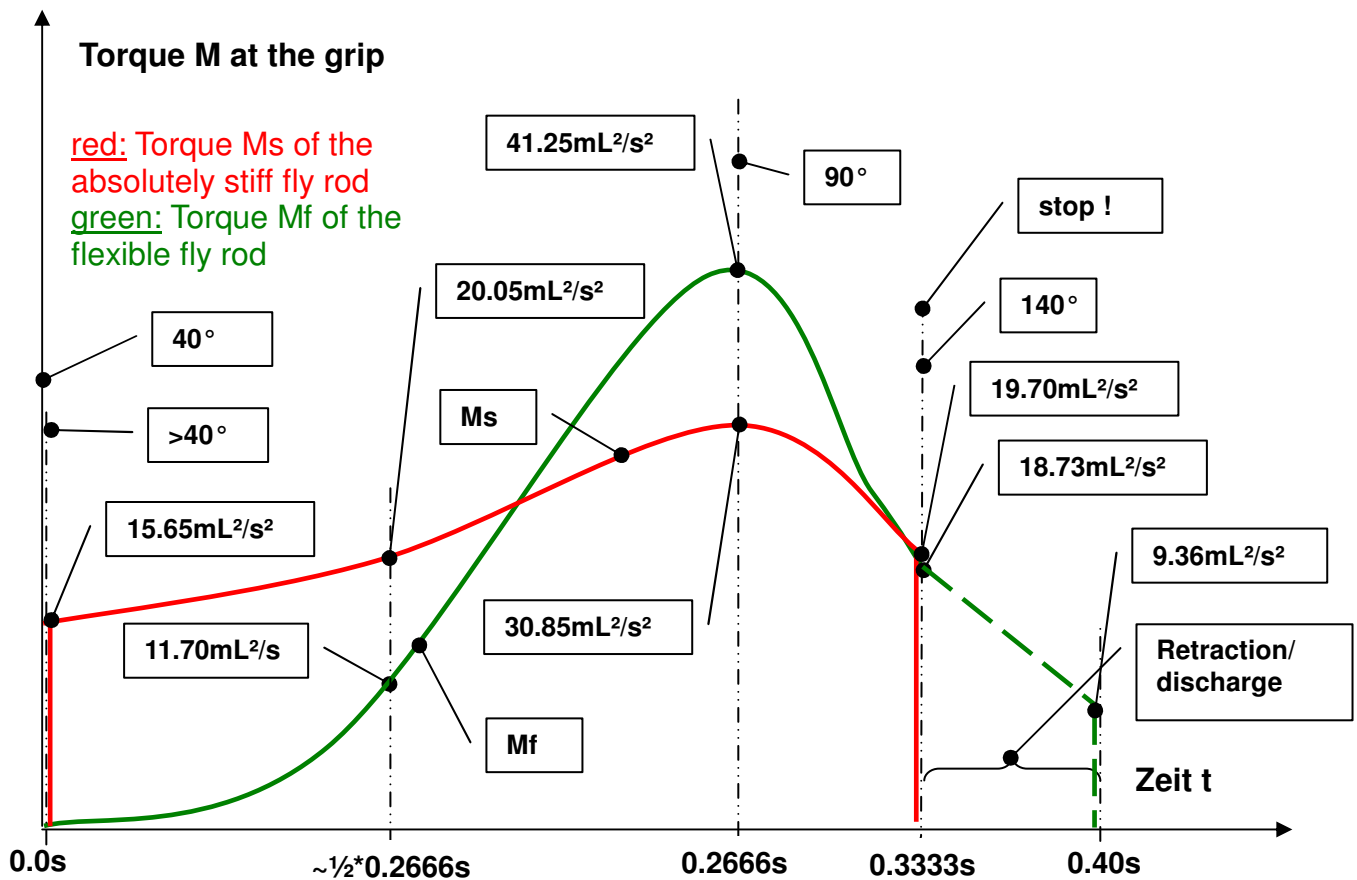


Figure XI

E) Efficiency

The ratio of the used energy and the spent energy is called efficiency or effectiveness η , respectively.

E1) Energy of the rod tips (efficiency)

The energy E, the tips of the flexible and the absolutely stiff fly rods reach at the end of the fly cast, correspond with the usable energy.²³ This energy can be calculated by the formula:

$$E = \frac{1}{2} * m * v(\text{end})^2$$

²³ In the following only the energies of the fly rod tips are respected. It is beyond the scope of this study to analyse how far these energies can be attributed to the fishing line. It might be a topic of future works. Surely it also depends on the fly caster's ability, to transfer the velocity of the fly rod tips onto the fishing line in an optimized fashion. Basically the maximized horizontal velocity at the end delivers the biggest potential for the highest speed of the fishing line.

The final velocities $v_f(\text{end})$ and $v_s(\text{end})$, respectively, of the tips of both fly rods have been calculated in section C3).

For the flexible fly rod it follows:

$$E_f = \frac{1}{2} * m * 12.535^2 \frac{L^2}{s^2} = \mathbf{78.56} \frac{m * L^2}{s^2}$$

For the absolutely stiff fly rod it follows:

$$E_s = \frac{1}{2} * m * 9.427^2 \frac{L^2}{s^2} = \mathbf{44.43} \frac{m * L^2}{s^2}$$

E2) Work, energy, respectively of the rotation (effort)

The work which has to be spent by the fly caster for the rotation of the fly rod, corresponds to the effort. It is calculated by the formula

$$A = \sum M * \alpha(\text{rad})$$

The angle $\alpha(\text{deg})$ measures 100° for both rods, corresponding to a radian measure (rad) of

$$\alpha(\text{rad}) = \pi * \frac{\alpha(\text{deg})}{180^\circ} = \pi * 0,555 = \mathbf{1.75}$$

If the grip is in the final position (140° position), then the fly caster has spent the whole work or energy, respectively, for the fly cast. During the retraction / discharge the rotational angle does not change any more and therefore the fly caster does not spend any further work / energy.

12. Conclusion: During the retraction/discharge of the flexible fly rod the fly caster does not spend any more work/ energy, because the rotational angle does not change ($\alpha=0$). During the retraction/discharge only the flexible fly rod is working while increasing the velocity of it's tip.

The sum of the torques in the rod grip working during the path length of the fly rod tip is well approximated by the trapezoidal formula in the following²⁴.

²⁴ More exact trapezoidal rule. http://en.wikipedia.org/wiki/Trapezoidal_rule

sum $M_f(t)$ of the torques of the flexible fly rod:

$$\begin{aligned} \sum M_f(t) &= \frac{1}{2} (M_0 + M_1) * \frac{1}{2} * 0.2666s && \sim 40^\circ - 65^\circ \\ &+ \frac{1}{2} (M_1 + M_2) * \frac{1}{2} * 0.2666s && \sim 65^\circ - 90^\circ \\ &+ \frac{1}{2} (M_2 + M_3) * \frac{1}{2} (0.3333s - 0.2666s) && \sim 90^\circ - 115^\circ \\ &+ \frac{1}{2} (M_3 + M_4) * \frac{1}{2} (0.3333s - 0.2666s) && \sim 115^\circ - 140^\circ \\ &+ \frac{1}{2} (M_4 + M_5) * (0.4s - 0.3333s) && \text{retraction/ discharge} \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} * 11.70 * 0.1333 + \frac{1}{2} * 52.95 * 0.1333 + \frac{1}{2} * 71.27 * 0.0333 \\ &+ \frac{1}{2} * 48.75 * 0.0333 + \frac{1}{2} * 28.09 * 0.0666 \end{aligned}$$

This value becomes zero after multiplication
with $\alpha=0$, see 12. conclusion

$$= 0.779 + 3.529 + 1.186 + 0.81 + 0.00 = \mathbf{6.30} \frac{m * L^2}{s^2} * s$$

$$A_f(t) = \sum M_f(t) * \alpha(\text{rad}) = 6.30 * 1.75 = \mathbf{11.025} \frac{m * L^2}{s^2} * s$$

Sum $M_s(t)$ of the torques of the absolutely stiff fly rod:

$$\begin{aligned} \sum M_s(t) &= \frac{1}{2} (M_0 + M_1) * \frac{1}{2} * 0.2666s && \sim 40^\circ - 65^\circ \\ &+ \frac{1}{2} (M_1 + M_2) * \frac{1}{2} * 0.2666s && \sim 65^\circ - 90^\circ \\ &+ \frac{1}{2} (M_2 + M_3) * \frac{1}{2} * (0.3333s - 0.2666s) && \sim 90^\circ - 115^\circ \\ &+ \frac{1}{2} (M_3 + M_4) * \frac{1}{2} * (0.3333s - 0.2666s) && \sim 115^\circ - 140^\circ \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} * 35.7 * 0.1333 + \frac{1}{2} * 50.90 * 0.1333 \\ &+ \frac{1}{2} * 56.10 * 0.0333 + \frac{1}{2} * 44.95 * 0.0333 \end{aligned}$$

$$= 2.672 + 3.392 + 0.934 + 0.748 = \mathbf{7.74} \frac{m * L^2}{s^2} * s$$

$$A_s(t) = \sum M_s(t) * \alpha(\text{rad}) = 7.74 * 1.75 = \mathbf{13.545} \frac{m * L^2}{s^2} * s$$

During the whole duration of the fly cast the following work / energy is spent at the rod grip:

$$A_f = A_f(t) / t = 11.025 / 0.3333s = \mathbf{33.07} \frac{m * L^2}{s^2}$$

$$A_s = A_s(t) / t = 13.545 / 0.3333s = \mathbf{40.67} \frac{m * L^2}{s^2}$$

E3) Efficiency of the flexible and the absolutely stiff fly rod

The direct comparison of the efficiency of both rods, which are cast under the same constraints delivers the following:

$$\eta = E(\text{rod tip}) / A(\text{rotation})$$

$$\eta_f(\text{flexible fly rod}) = E_f / A_f = \frac{78,56}{33,07} = \mathbf{2.37}$$

$$\eta_s(\text{absolutely stiff fly rod}) = E_s / A_s = \frac{44,43}{40,67} = \mathbf{1.09}$$

According to the initially stated constraints to both rods no mass is contributed. This constraint is the reason that the calculated efficiencies are higher than 1.0²⁵. As the negligence of the mass has the same result for both rods, the comparison of the efficiencies η_f/η_s is still valid.

$$\eta_f / \eta_s = 2.37 / 1.09 = 2.17$$

13. Conclusion: The efficiency of the flexible fly rod is more than twice (factor 2.17) the efficiency of the absolutely stiff fly rod.

The work/ energy that has to be spent by the fly caster also depends on the mass of the fly rod.

14. Conclusion: As less weight / mass the fly rod carries, as stronger it's efficiency increases (keeping all other properties of the fly rod unchanged).

E4) Change of the efficiency during changing deflection

I would like to estimate shortly, how the efficiency changes with more or less deflection of the flexible fly rod. This analysis is only a rough estimation and does not intend to substitute a more exact calculation.

²⁵ The efficiency η can not be bigger than 1.0, as the gained energy can never be bigger than the effort. When respecting the mass of both rods the torques in the grip would rise strongly in comparison to the calculated values, as that the efficiencies of both fly rods would be smaller than 1.0. The following estimation suggests: Assuming a weight of both fly rods of about 80 grams (which corresponds to the weight of the used fly rod SAGE 586 RPL+) and a weight of the elongated fishing line of about 10 grams (which corresponds to the weight of the used fishing line WF5F) leads to a factor $80g/10g = 8$. If this mass ratio is approximately added to the force acting on the tip, also the moments as well as the effort work/energy rises by this factor. Then the efficiencies of both fly rods are below 1.0

From my preceding investigations a good estimation can be gathered for the case, that the deflection of the flexible fly rod is smaller in comparison to the given one in my fly cast sequence. In such case the values calculated for my fly cast sequence become more similar to the values calculated for the absolutely stiff fly rod, it means, the efficiency decreases.

The case of a higher deflection than the one given in my fly cast sequence is difficult to analyse due to lacking calculated values and due to the fact that my created deflection is quite big – in comparison to the fly cast sequences of other fly casters²⁶.

Surely the efficiency cannot be increased infinitely by rising deflection as it also depends on the possibility of the given fly rod to retract²⁷. If the retraction capacity of the fly rod reached a certain limit, further increase of the effort (work) most probably would not show a visible bigger utility (final velocity).

At this point I would like to make a comparison to pole vault²⁸: before 1900, when virtually stiff poles from the ash tree were used, the jumped height was limited to about 3 meters. With modern, flexible poles, about twice the height can be vaulted. The prerequisite for this, however, is that the deflection of the pole is well fitted to the weight (mass) of the pole jumper and his attempt velocity. Also this example shows, that flexible poles outmatch the stiff ones by far on the one hand, but also the deflection properties (retraction capacity) of the flexible pole plays an important role on the other hand.

On closer examination of my investigations I see an important reason for the efficiency of the flexible fly rod given by the fact that the work, the force, respectively, at the grip is spent by a short pulse "peak" in comparison to the absolutely stiff rod. This in turn, leads to a strong deflection of the flexible fly rod²⁹.

As long as the increasing deflection leads to an increase of this short pulse „peak“ of torque **and** an increase of the final velocity, as long the efficiency rises.

This estimation shows, that there is a correlation between the deflection of the fly rod and it's efficiency.

The following Figure XII shows – roughly estimated – the changes of the work at the rod grip during changing deflection:

²⁶ E.g. in comparison to the investigations of Løvøll & Jason Berger from 2007 – ‚The Rod & The Cast‘. The deflection of my fly rod is much bigger than the deflection of the one investigated there.

²⁷ As faster a fly rod retracts into linear position as bigger the retraction potential is.

²⁸ As for other comparisons it also accounts for this one: Pole vault cannot and should not be equal to fly casting, because there are differences for sure. But there are parallel properties well picturing the impact of the efficiency during fly casting.

²⁹ To gain a short pulse of torque, the acceleration of the rod tip needs to be high for a short term. An essential prerequisite for this is a preferably small acceleration of the rod tip at the beginning of the fly cast. Therefore it is important, to initiate the fly cast slowly and to increase it towards the end. A more than linearly (nonlinear) rising acceleration of the rod tip is an advantage in such case.

acceleration of the tip, i.e. the velocity rises strongly.

The angular momentum is based on the energy of rotation³¹ and calculates according to the following formula:

$$D = m * r^2 * \omega; \text{ mit } \omega = \frac{v(\text{rodtip})}{r} \rightarrow D = m * r^2 * \frac{v}{r} = m * r * v$$

with D = Angular momentum; m = rotating mass; r = radius between center of rotation and the tip of the rod; ω = angular speed around the center of rotation

The radius r has to be rectangular to the horizontal velocity (index h).

$$D = m * r, h^2 * \frac{v, h}{r, h} = m * r, h * v, h$$

The solution of the previous formula for v,h denotes to: $v, h = \frac{D}{m * r, h}$

The apex of the deflection of the fly rod is a suitable approximation for the center of rotation and it is marked with the symbol \odot in the following Figure XIII³².

The angular momentum D is a physical invariant and has to be constant during the fly cast. It remains in the system when the center of rotation is shifted. Therefore the angular momentum increases the velocity of the tip when the center of rotation is shifted upwards because the radius between center of rotation and tip shortens. For a simplified investigation we can assume, the angular momentum would be generally constant during the whole duration of the cast:

$$D(40^\circ) = D(90^\circ) = D(140^\circ) = D(\text{discharge}) = \text{constant}$$

³¹ Energy of movement, therefore kinetic energy, http://en.wikipedia.org/wiki/Angular_momentum

³² A more detailed determination of the center of rotation is very complicated and only possible with higher mathematics. For the presentation of the mode of action of the conservation of angular momentum the described approximation is sufficient.

For the flexible fly rod it follows:

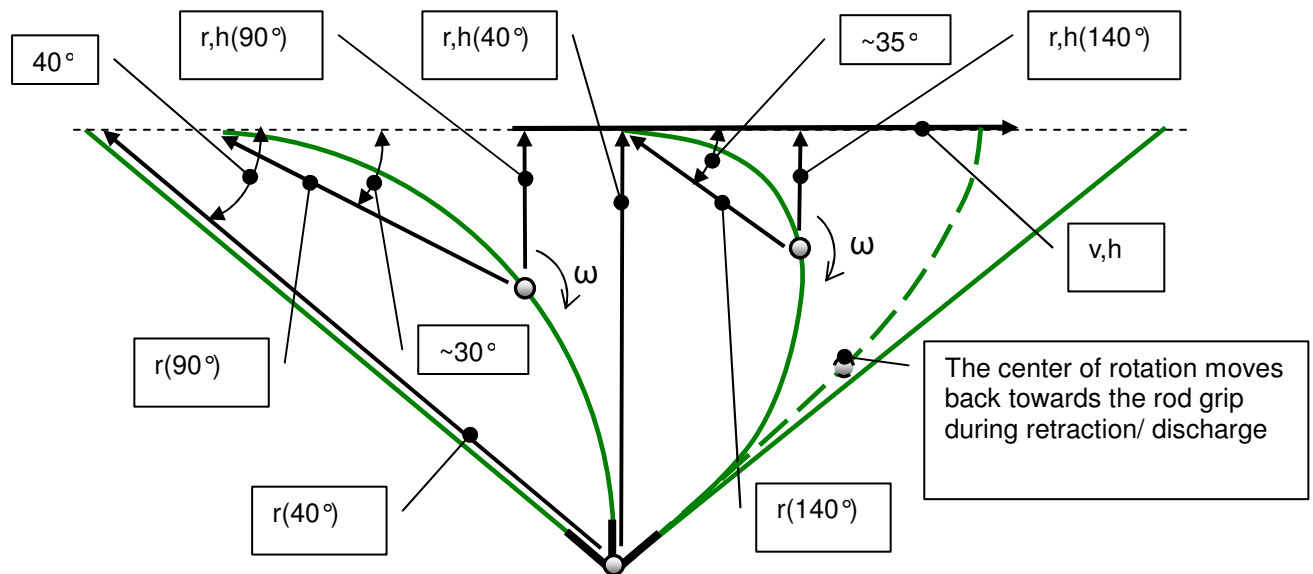


Figure XIII

The geometrical properties are determined from the previous Figure XIII³³. The radius r can be approximated by:

$$r(40^\circ) \sim L; r(90^\circ) \sim \frac{1}{2} L; r(140^\circ) \sim \frac{1}{3} L$$

The radius r,h rectangular to the velocity is calculated in the following table:

Position rotational angle/ position of the rod grip	40°	90°	140°
Radius r of the angular momentum	$\sim L$	$\sim \frac{1}{2} L$	$\sim \frac{1}{3} L$
Radius r,h of the angular momentum rectangular to the velocity v,h	$r,h \approx r(40) * \sin(40) = 0.643 L$	$r,h \approx r(90) * \sin(30) = 0.250 L$	$r,h \approx r(140) * \sin(35) = 0.191 L$

The moving mass m is located above the center of rotation at most. As the center of rotation moves towards the tip with increasing deflection in my fly cast sequence, the radius shortens and the moving mass reduces. For the moving mass m the following can be stated:

³³ A more detailed determination of the geometrical relations is elaborate and would be beyond the scope of my investigations. This section shall elucidate the principal impact of the conservation of angular momentum. For that purpose the approximated geometrical relations are sufficient.

$$m(40^\circ) > m(90^\circ) > m(140^\circ)$$

For the sake of simplicity this positive effect of the reduction of the moving mass m is neglected and it is assumed that the mass is constant. The angular momentum acts onto the horizontal velocity v,h of the rod tip as follows:

$$v,h(>40^\circ) = \frac{D}{m * 0.643L} ; v,h(90^\circ) = \frac{D}{m * 0.25L} ;$$

$$v,h(140^\circ) = \frac{D}{m * 0.191L} ; v(\text{discharge}) = \frac{D}{m * 0.643L}$$

$$v,h(90^\circ)/v,h(>40^\circ) = \frac{0.643}{0.25} = \mathbf{2.57} ; v,h(140^\circ)/v,h(90^\circ) = \frac{0.25}{0.191} = \mathbf{1.31}$$

$$v(\text{discharge})/v,h(140^\circ) = \frac{0.191}{0.643} = \mathbf{0.30}$$

15. Conclusion: The impact of the angular momentum alone leads to an increase of the horizontal velocity of the rod tip by a factor of 2.57 between the 40° and the 90° position and another additional factor 1.31 between the 90° and the 140° position. Especially when the deflection increases, the horizontal velocity increases significantly. The decrease of this velocity (according to the conservation of angular momentum) by a factor of 0.30 during the retraction/discharge is compensated by the initiating tension release of the fly rod (spring effect)³⁴.

For the absolutely stiff fly rod this effect of the angular momentum does not exist because it does not deflect. It's center of rotation stays in the rod grip.

For a very stiff fly rod which deflects only a little, the center of rotation moves a bit from the grip towards the rod tip, however it does not reach the large distances from the grip like in my fly cast sequence. Therefore in such case the decrease of the radius r and r,h , respectively, is smaller as also the increase of the velocity due to the conservation of angular momentum.

The effect of the conservation of angular momentum is in agreement with my estimation in section E4), that the efficiency reduces when the deflection decreases. Moreover the investigation shows that the increasing deflection until the initiation of the stop, the

³⁴ Due to the previously described, simplified assumptions the factors do not claim to represent exactly the increase of the velocity of the rod tip. The interplay between the introduced rotational energy, the shift of the center of rotation and the potential tension energy leads to the fact that the tip of the flexible fly rod moves slower than the tip of the absolutely stiff fly rod but starts to catch up due to the shift of the center of rotation in spite of the further rise of the potential tension energy. Already in the 90° position the tip of the flexible fly rod reaches the speed of the tip of the absolutely stiff fly rod. Beyond this position the speed of the tip of the flexible fly rod passes the speed of the absolutely stiff fly rod and finally reaches a value 33 % higher than the speed of the absolutely stiff fly rod (see 7. conclusion). For the flexible fly rod most of the energy is transferred at the end of the fly cast.

retraction/discharge not only increases the potential tension energy of the fly rod but also strengthens the effect resulting from the conservation of angular momentum³⁵.

F2) Influence of the parallel translation

The distance of the position of the rod grip between the stop positions (initial and final position) indicates, how strongly the parallel translation is applied by the fly caster. As bigger as this distance is, as more the parallel translation is inserted into the fly cast. It prolongates the pathway, the rod tip propagates – in comparison to it's pathway during exclusive rotational movement.

F2.1) Parallel translation in the fly cast sequence

From my fly cast sequence it becomes visible, that I infer a comparatively long parallel translation (see pictures in section C2). It measures about one meter³⁶. The pathway length of the rod tip prolongates by this one meter. The rod tip moves along a distance of $1.54 * L$ (see section B3) due to the rotational movement.

At a length of 2.65m, as it is the case for he used SAGE 586 RPL+, this rotational movement measures to:

$$WRs(f) = 1.54 * 2.65m = 4.08 \text{ m}$$

Therefore the proportion of the parallel translation in my casting sequence is $1.0m / 4.08m \approx 1/4$ of the whole pathway of the rod tip. It attracts attention that this proportion decreases for longer fly rods and increases for shorter fly rods, respectively.

16. Conclusion: As shorter the fly rod is, as more the parallel translation contributes to the total, horizontal path of the fly rod tip. As longer the fly rod is, as less the parallel translation contributes to the total, horizontal path of the fly rod tip³⁷.

From my casting sequence it becomes evident furthermore, that the parallel translation is conducted mainly at the beginning of the fly cast. Therefore a possibly existing part of a "slack fishing line" at the beginning of the fly cast is eliminated, the fishing line is connected linearly to the rod tip ("force coupling") and the fly rod is preloaded for the subsequent rotational movement (see pictures 1 to 6 in section C2). The deflection of the

³⁵ The fact that the deflection increases until the initiation of the stop, the retraction/discharge, was pointed out by the 9. conclusion. As long as the deflection of the fly rod increases in the same way as stated in my fly cast sequence, also the radius shortens. The shortening of the radius increases the effect of the conservation of angular momentum. The investigation of the angular momentum shows, that an increasing deflection of the fly rod can be advantageous for the fly caster.

³⁶ The parallel translation is estimated from the distance of the rod grip between the first and the last picture of my fly casting sequence. The reference is given by my body height of 1.77 m.

³⁷ Hereby it also becomes clear, that a pronounced parallel translation is a big advantage for short fly rods. "Short sticks" (Fly rods around 2 meters) therefore are thrown „from the shoulder“ with inactive wrist - see Hans Gebetsroither. Therefore the rotational center shifts downwards and the proportion of the parallel translation increases – by this the path of the rod tip can increase significantly especially for those short fly rods.

fly rod is activated directly at the beginning of the fly cast and can be increased smoothly³⁸.

This additional path from the parallel translation delivers advantages for the fly caster. The fly caster of the absolutely stiff fly rod could move along a linear part of the path of the rod tip employing the parallel translation. The parallel translation also secures, that the center of rotation of the fly cast is located at a lower point. (see following Figure XIV). As lower the center of rotation as more the linear part of the path of the rod tip can be increased while keeping small the rotational angle. This is a big advantage for a narrow loop.

As the translational shift operates similar for both rods it is inferred that it's influence annihilates when doing a comparison.

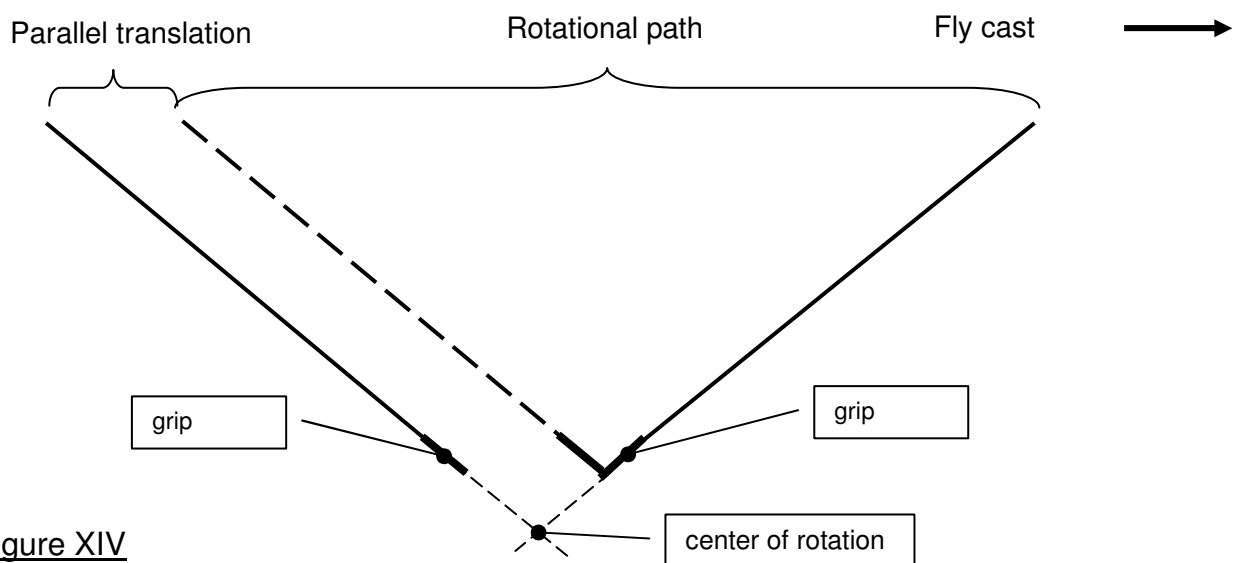


Figure XIV

F2.2) Pure parallel translation for the absolutely stiff fly rod

According to the initially stated prerequisites I have assumed a rotational movement for the absolutely stiff fly rod. From this follows a significant disadvantage according to the decomposition of the forces I described in chapter B4)³⁹. In the following I want to estimate shortly, if the absolutely stiff fly rod could have become more efficient, if this disadvantage would be erased.

The described disadvantage would be omitted, if the absolutely stiff fly rod could be cast with a pure parallel translation. The pure parallel translation is shown in Figure XV in comparison to the investigated rotational movement.

³⁸ For that reason also the efficiency of other special fly castes like e.g. the „rolling fly cast“ ("half fly cast") or Switch Cast (also named D- Cast) can be increased significantly by a pronounced parallel translation.

³⁹ On the other hand the absolutely stiff fly rod gains an advantage from the rotational movement that the lever arm shortens and therefore the effort for the fly caster reduces (see sections D3 and E2).

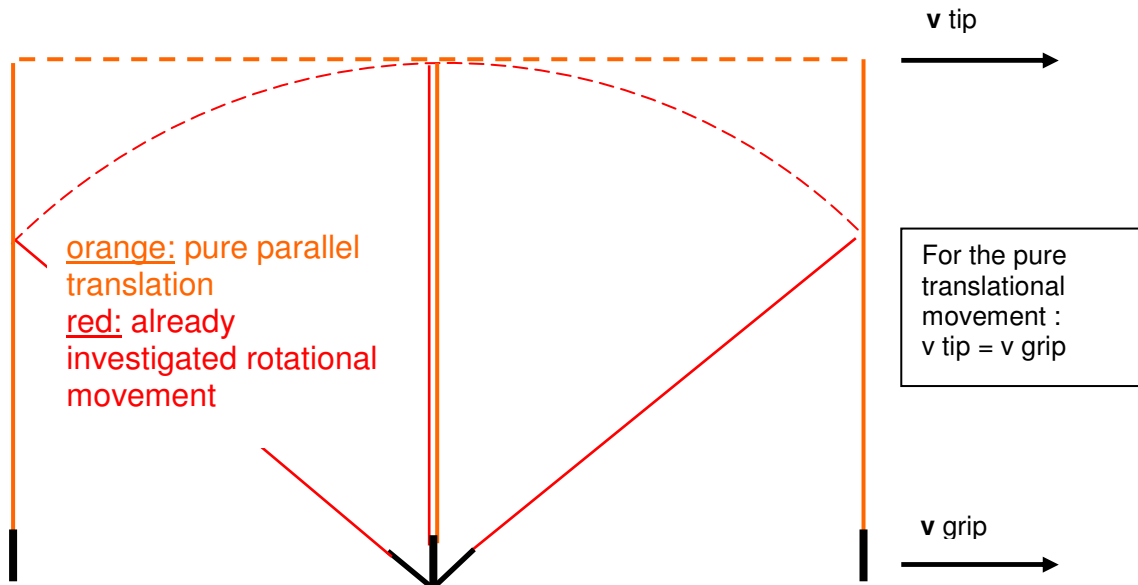


Figure XV

In such case the velocities of the tip and the grip of the absolutely stiff fly rod would be equal. According to the relation $E = \frac{1}{2} * m * v^2$, the equal velocities create the same energy. Therefore the energy which is transferred into the grip by the fly caster must be the same like the energy which is transferred to the rod tip:

$$\eta(\text{translation}) = E(\text{rod tip}) / E(\text{grip}) = 1.0$$

With 1.09 the efficiency calculated in section E3) for the rotational movement of the absolutely stiff fly rod is virtually the same. At parallel translation only the efficiency of the absolutely stiff fly rod would not be higher than at a rotational movement, I proposed in my investigations and compared with the results of the flexible fly rod.

The absolutely stiff fly rod neither is more efficient than the flexible fly rod in my fly cast sequence when moved by parallel translation only nor when moved rotational only.

17. Conclusion: Neither by pure rotational nor by pure parallel translational movement the efficiency of the absolutely stiff fly rod can be increased. There are no signs that this changes at a combination of both fly cast techniques. Due to the rotational movement proposed for my investigations the absolutely stiff fly rod does not get disadvantages which would question the conclusions of my investigations.

G) Conclusion and final investigations

The results are related to an analysis of my fly cast sequence. It remains open if an analysis of other fly castes would deliver similar results.

Final Conclusion: My investigations show, that the deflection of the fly rod leads to advantages during the whole duration of the fly cast. These advantages result mainly from the interplay between the conservation of angular momentum and the spring tension energy. The parallel translation can help to use these effects better. According to the missing deflection the absolutely stiff fly rod does not show such advantages. The flexible fly rod transfers the energy which is inferred into the grip much more efficient into the tip than the absolutely stiff fly rod can do.

G1) Uncertainties

The most of my initially stated boundary conditions should lead to the same uncertainties for both rods. The conclusions drawn from the boundary conditions resulted from a comparison / a quotient and therefore neutralize each other. Therefore the conclusions do not need to be questioned.

The uncertainties resulting from the fact that only three positions of the fly rod were investigated and the values in between were interpolated are more difficult to judge. This might lead to inappropriate results especially for the flexible fly rod because it's behavior cannot be described by a linear course at most. Surely my investigation would have been more exact if further positions would have been evaluated but this would have been beyond the scope of this study and beyond my technical options. At this point I have to suppose that the investigated positions are characteristic to image the reality. This is supported by the fact that my results are widely consistent.

Even if the calculated values would shift disadvantageously for the flexible fly rod at more exact evaluation the shown advantages in comparison to the absolutely stiff fly rod would remain.

G2) Conclusion

From my investigations mainly the following advantages result, a flexible fly rod has in comparison to an absolutely stiff fly rod:

- The final horizontal speed of the tip of the flexible fly rod is significantly larger than for the absolutely stiff fly rod.

- The fly caster can initially slowly increase the force, the work, respectively for the flexible fly rod. At the end of the fly cast he can decrease it. The absolutely stiff fly rod needs a prompt effort of force, work, respectively, promptly decaying at the end of the fly cast.
- The deflection of the flexible fly rod shows two positive effects via the angular momentum and the spring tension, which are not found for the absolutely stiff fly rod
- For the flexible fly rod the most intense force, work, respectively, needs to be spent only for a very short time period („peak“). For the absolutely stiff fly rod a quite slowly changing force has to be spent during the whole duration.
- ***The efficiency of the flexible fly rod is more than twice the efficiency of the absolutely stiff fly rod.***

If only the flexible fly rod is analyzed solely, many advantages offered the fly caster by the deflection are not clearly visible. Just the comparison between the flexible and the absolutely stiff fly rod show the advantages of the flexible fly rod. Without doubts also the absolutely stiff fly rod can cast a fishing line. But the fly caster has to spend more force, work, respectively for the same end velocity of the rod tip.

Especially due to the properties of the deflection the flexible fly rod can increase the efficiency in comparison to the absolutely stiff fly rod. It is important that the weight of the fishing line and the desired final speed is adapted to the retraction potential of the fly rod.

As better the caster uses the deflection of the fly rod in the range of its capacity, as more efficient the fly rod transfers the work introduced by the caster into its tip⁴⁰.

Acknowledgements and final remarks

My gratitude belongs to

- a) Especially the physicist Dr. Franz-Josef Schmitt, who had advised and supported me regarding all physical context,
- b) The certified Flycasting instructor and mathematician Dr. sc. math. Jean-Paul Kauthen, who has given me precious advises for the whole duration of elaboration.

Without your elaborated help my investigations could not have been presented with such precise formulations.

Finally I thank my meanwhile nearly three years old twin daughters Greta & Theresa. Because you sleep the whole night since your 4th month I was able to find the necessary time and concentration in the evening for my investigations. I hug you !

⁴⁰ My investigations underline and prove the statements of the teaching film „perfect fly casting“ (German:“ Perfektes Fliegenwerfen“), which was produced by H.R. Hebeisen together with me in 2009. By the way Dr. Schmitt (see acknowledgements) made the remark, that evolution always leads to the survival of the best technique if it is used often. My investigation present the essential reasons why we fly fishers work with the flexible instead of the absolutely stiff fly rod!

Altogether with interrupted time slots I worked more than one year on these investigations until I finalized them in early February 2014. During this time span I had some „Eureka“-effects that taught me the physical requirements for an efficient, force minimizing fly cast. These insights will be incorporated into my work as a certified Flycasting instructor !

I am aware that fly casting is a very complex physical problem when treated more exactly. It was my main aim to have a short look on the physical values and effects which act on the fly rod during the cast. I would be pleased if my investigations could contribute, to better understand the physical context of fly casting.

Potsdam-Rehbrücke in February 2014

Tobias Hinzmann

Annex

Summary of the conclusions:

1. Conclusion: *The deflection of the fly rod leads to a dynamics of the rod tip moving along a straight line.*

2. Conclusion: *The deflection of the fly rod leads to a shortened projection onto the vertical 90° position by about 1/3 (0.36*L).*

3. Conclusion: *The tip of the flexible fly rod points into the direction of the elongated fishing line for a huge part of the path; it accelerates the fishing line in the horizontal direction. The tip of the absolutely stiff fly rod does not point into the direction of the elongated fishing line at any time; it accelerates the fishing line in the tangential direction along the segment of a circle.*

4. Conclusion: *After a rotational angle α of 100° the horizontal part of the velocity of the tip reduces by a factor of 0.643 for the absolutely stiff fly rod in the final position. This factor reduces with rising rotational angle α and therefore the horizontal part of the velocity that can be reached with the absolutely stiff fly rod.*

5. Conclusion: *The stop, the retraction/discharge, respectively, of the flexible fly rod starts, when it's tip has completed about the first half (~50 %) of the path length (rotational path). The rod hand then has reached the final position at 140°.*

6. conclusion: *The tips of both rods take different time periods, to reach the final horizontal velocity. At the same rotational speed the fly cast of the flexible fly rod takes about 0.0666 seconds longer (the time period of the retraction/discharge) than the fly cast of the absolutely stiff fly rod.*

7. Conclusion: *The tip of the fly rod has a final horizontal velocity which is about 33 % higher in comparison to the absolutely stiff fly rod. In marked contrast the tip of the flexible fly rod has only 15 % of the initial velocity in comparison to the absolutely stiff fly rod at the beginning of the fly cast (initial velocity).*

8. Conclusion: *The tip of the flexible fly rod has more than 80 % of it's final velocity at the beginning of the stop/ retraction/ discharge.*

9. Conclusion: *The displacement measures the deflection of the fly rod. The displacement increases for more than until the 90° position is reached. The whole displacement introduced into the rotation (change of rotational angle) increases the deflection of the fly rod.*

10. Conclusion: *The initial rotation leads to increasing deflection for the flexible fly rod causing an increase of the potential tension force without an acceleration of the mass of the fishing line. For the absolutely stiff fly rod each initial movement immediately leads to an acceleration of the fishing line and the immediate need to spend force for it.*

11. Conclusion: *The torque which has to be spent by the fly caster rises with the square of the length L of the fly rod.*

12. Conclusion: During the retraction/discharge of the flexible fly rod the fly caster does not spend any more work/ energy, because the rotational angle does not change ($\alpha=0$). During the retraction/discharge only the flexible fly rod is working while increasing the velocity of it's tip.

13. Conclusion: The efficiency of the flexible fly rod is more than twice (factor 2.17) the efficiency of the absolutely stiff fly rod.

14. Conclusion: As less weight / mass the fly rod carries, as stronger it's efficiency increases (keeping all other properties of the fly rod unchanged).

15. Conclusion: The impact of the angular momentum alone leads to an increase of the horizontal velocity of the rod tip by a factor of 2.57 between the 40° and the 90° position and another additional factor 1.31 between the 90° and the 140° position. Especially when the deflection increases, the horizontal velocity increases significantly. The decrease of this velocity (according to the conservation of angular momentum) by a factor of 0.30 during the retraction/discharge is compensated by the initiating tension release of the fly rod (spring effect).

16. Conclusion: As shorter the fly rod is, as more the parallel translation contributes to the total, horizontal path of the fly rod tip. As longer the fly rod is, as less the parallel translation contributes to the total, horizontal path of the fly rod tip.

17. Conclusion: Neither by pure rotational nor by pure parallel translational movement the efficiency of the absolutely stiff fly rod can be increased. There are no signs that this changes at a combination of both fly cast techniques. Due to the rotational movement proposed for my investigations the absolutely stiff fly rod does not get disadvantages which would question the conclusions of my investigations.

Final Conclusion: My investigations show, that the deflection of the fly rod leads to advantages during the whole duration of the fly cast. These advantages result mainly from the interplay between the conservation of angular momentum and the spring tension energy. The parallel translation can help to use these effects better. According to the missing deflection the absolutely stiff fly rod does not show such advantages. The flexible fly rod transfers the energy which is inferred into the grip much more efficient into the tip than the absolutely stiff fly rod can do.